

Entrywise preservers beyond Schoenberg

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Abstract:

The Schur product theorem states that the Hadamard product, given by multiplying matrices entrywise, preserves the property of being positive semidefinite.

It follows immediately that applying any absolutely monotonic function entrywise also preserves this form of positivity, where a function is said to be absolutely monotonic if its Maclaurin series has non-negative coefficients.

Schoenberg showed that the converse is true: a function that preserves positive semidefiniteness when applied entrywise to matrices of arbitrary size is necessarily absolutely monotonic.

The situation is more complex for matrices of a fixed size, or when the class of matrices being studied has some other form of positivity constraint, or possesses additional structure, such as Hankel or Toeplitz matrices.

Recently characterisations have been obtained for the class of entrywise maps preserving the inertia of real symmetric matrices (that is, the total multiplicity of positive, zero and negative eigenvalues) and for entrywise transforms that satisfy various negativity constraints.

This talk will provide a leisurely tour of these ideas and results, based on joint work with Dominique Guillot (University of Delaware), Apoorva Khare (Indian Institute of Science, Bangalore) and Mihai Putinar (University of California at Santa Barbara and Newcastle University).