Curvature regulates wrinkling patterns on surfaces

Fan Xu

Fudan University, Shanghai 200433, P.R. China

Email: fanxu@fudan.edu.cn

Biological functions in living systems are closely related to their geometries and morphologies [1-4]. Many natural structures present interesting features containing positive, zero, and negative Gaussian curvatures within one system. Such varying curvatures would significantly affect the growing or dehydrating morphogenesis, as observed in various intricate patterns in abundant biological structures (see Fig. 1). To understand the underlying morphoelastic mechanism and to determine the crucial factors that govern the patterning in structures with variable curvatures, we develop core-shell models and derive scaling laws to characterize growth- or dehydration-induced instability patterns. We find that the eventual patterns are mainly determined by dimensionless parameters that are composed of stiffness and curvature of the system. Moreover, we construct phase diagrams showing the multiphase wrinkling pattern selection in various structures in terms of the key parameters, which is confirmed by our experimental observations. Physical insights into the multiphase transitions and existence of bistable modes are further provided by identifying hysteresis loops and the Maxwell equal-energy conditions. The universal law for morphology selection on core-shell structures with varying curvatures can fundamentally explain and precisely predict wrinkling patterns of diverse structures, which may also provide a platform to design morphology-related functional surfaces.

Fig. 1. Curvature regulates wrinkling patterns in diverse biological structures.

Acknowledgements: This work is supported by the National Natural Science Foundation of China (Grants No. 12122204 and 12372096), Shanghai Shuguang Program (Grant No. 21SG05), and Shanghai Pilot Program for Basic Research-Fudan University (Grant No. 21TQ1400100-21TQ010).

References:

[1] T. Wang, Z. Dai, M. Potier-Ferry, F. Xu. Curvature-regulated multiphase patterns in tori. *Physical Review Letters*, 130, 048201, 2023.

[2] F. Xu, Y. Huang, S. Zhao, X.Q. Feng. Chiral topographic instability in shrinking spheres. *Nature Computational Science*, 2, 632–640, 2022.

[3] F. Xu, S. Zhao, C. Lu, M. Potier-Ferry. Pattern selection in core-shell spheres. *Journal of the Mechanics and Physics of Solids*, 137, 103892, 2020.

[4] T. Wang, Y. Yang, C. Fu, F. Liu, K. Wang, F. Xu. Wrinkling and smoothing of a soft shell. *Journal of the Mechanics and Physics of Solids*, 134, 103738, 2020.