Nonlinear Anisotropic Viscoelasticity

Souhayl Sadik[1] and Arash Yavari[2,3]

[1] *Department of Mechanical and Production Engineering, Aarhus University, 8000 Aarhus C, Denmark*
[2] *School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA 30332, USA*
[3] *The George W. Woodruff School of Mechanical Engineering, Georgia Institute of Technology, Atlanta, GA 30332, USA*

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Abstract

In this work, we revisit the mathematical foundations of nonlinear viscoelasticity within the framework of geometric nonlinear elasticity. We study the kinematics and the underlying geometry of viscoelastic deformations, and in particular, the so-called intermediate configuration which we find in fact to be fundamentally different for viscoelasticity compared to anelasticity. Starting with the multiplicative decomposition of the deformation gradient $\mathbf{F} = \overset{e}{\mathbf{F}}\overset{v}{\mathbf{F}}$, we show based on physical arguments that $\overset{v}{\mathbf{F}}$ ought to be a material tensor and $\overset{e}{\mathbf{F}}$ a two-point tensor. It is assumed that the free energy density is the sum of an equilibrium and a non-equilibrium part; and a dissipation potential is assumed to drive the evolution of the viscous deformation. Following this two-potential approach, the governing equations (constitutive laws, balance laws, and kinetic equations) of nonlinear viscoelasticity are derived using the Lagrange-d'Alembert principle and the laws of thermodynamics. The symmetry transformations and their action on the total, elastic, and viscous deformation gradients are carefully discussed. We discuss the constitutive and kinetic equations for compressible and incompressible isotropic, transversely isotropic, orthotropic, and monoclinic viscoelastic solids. We discuss quadratic dissipation potentials and semi-analytically study creep and relaxation in three examples of universal deformations.

Keywords: Geometric mechanics, nonlinear viscoelasticity, multiplicative decomposition, intermediate configuration, anisotropic solids.