Asymptotic numerical method for hyperelasticity

Michel Potier-Ferry, LEM3, Université de Lorraine

Email : michel.potier-Ferry@univ-lorraine.fr

The asymptotic numerical method is a numerical continuation method based on Taylor series. Introduced a long time ago [1], it has been developed from the 1990s. The first ingredient is high-order differentiation technique for which there are well-established procedures in the literature [2]. When applied to a hyperelastic constitutive law, the differentiation provides numerically an affine relation between the Taylor coefficients of strain and stress tensors and it is easily coupled with the balance of momentum [3]. The CPU time of a power series is not very large because the same stiffness matrix is used for all the orders.

Knowledge of Taylor series is known to define an analytic function, even beyond the radius of convergence, which means that the knowledge of a series can provide useful information on the global response of the system. In this way, the radius of convergence of the exact series or the range of validity of the truncated series are easily obtained, which permits to simply establish a continuation method with a perfectly adaptive step length. Second, improved approximations of boundary value problems can be deduced from the series via acceleration techniques as Padé approximants or "extrapolation methods". Last bifurcation analyses were done by various techniques starting from these Taylor series.

There are many works using this continuation method, in particular for buckling of elastic structures. There are also some applications of the method to elastoplastic bodies, in fluid mechanics and for nonlinear vibration. This talks reviews theoretical aspects and various applications of this numerical method.

[1] Thompson, J. M. T., Walker, A. C. (1968). The non-linear perturbation analysis of discrete structural systems. *International Journal of Solids and Structures*, *4*, 757-768.

[2] Griewank, A., Walther, A. (2008). *Evaluating derivatives: principles and techniques of algorithmic differentiation*. Society for Industrial and Applied Mathematics.

[3] Nezamabadi, S., Zahrouni, H., Yvonnet, J. (2011). Solving hyperelastic material problems by asymptotic numerical method. *Computational mechanics*, *47*, 77-92.