Some Remarks on the Love hypothesis in Nonlinear Elasticity

Andrea Nobili[1], Giuseppe Saccomandi[2,3]

[1]*Department of Engineering "Enzo Ferrari", University of Modena and Reggio Emilia, 41125 Modena, Italy*
E-mail: andrea.nobili@unimore.it

[2]*Dipartimento di Ingegneria e Sezione INFN di Perugia, Università degli Studi di Perugia, 06125 Perugia, Italy*
E-mail: giuseppe.saccomandi@unipg.it

[3]*School of Mathematics, Statistics and Applied Mathematics, NUI Galway, University Road, Galway, Ireland*

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We consider the Love hypothesis, originally introduced to encompass for dispersion of longitudinal waves in linear elastic rod [3]. Despite belonging to the linear framework, this hypothesis is sometimes extended to the nonlinear regime [5, 2], where it clearly fails in the case of incompressible materials [1]. Accordingly, we re-examine the origin of the assumption and develop a refined version, which is asymptotically consistent and takes into account inertial effects. Indeed, the refined version of the Love hypothesis originates from a multiscale analysis, in the linear regime, of the well-known Mindlin-Herrmann two-modal setup [4], that may be easily generalized to the nonlinear framework. We then proceed to show how this refined assumption impact on the constraint of incompressibility, which proves instrumental in developing a nonlinear generalization.

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