Some Remarks on the Love hypothesis in Nonlinear Elasticity

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We consider the Love hypothesis, originally introduced to encompass for dispersion of longitudinal waves in linear elastic rod \cite{Love}. Despite belonging to the linear framework, this hypothesis is sometimes extended to the nonlinear regime \cite{Wright,Dai}, where it clearly fails in the case of incompressible materials \cite{Amendola}. Accordingly, we re-examine the origin of the assumption and develop a refined version, which is asymptotically consistent and takes into account inertial effects. Indeed, the refined version of the Love hypothesis originates from a multiscale analysis, in the linear regime, of the well-known Mindlin-Herrmann two-modal setup \cite{Mindlin}, that may be easily generalized to the nonlinear framework. We then proceed to show how this refined assumption impact on the constraint of incompressibility, which proves instrumental in developing a nonlinear generalization.

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References

\cite{Amendola} A Amendola and G Saccomandi. A simple remark about the love hypothesis in rod dynamics. Applications in Engineering Science, 8:100076, 2021.


\cite{Mindlin} Michael Shatalov, Julian Marais, Igor Fedotov, Michel Djouosseu Tenkam, and M Schmidt. Longitudinal vibration of isotropic solid rods: from classical to modern theories. Advances in computer science and engineering, 1877:408–9, 2011.