An incompability and stress-driven volumetric growth law

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Abstract

Soft tissue growth and remodelling are fundamental processes in various physiological applications, and mathematical modelling plays a crucial role in understanding disease prognoses. Volumetric growth theory is a commonly used mathematical approach in this context. Our previous research on volumetric growth theory focused on defining the growth tensor in the current (loaded) configuration, which revealed that it better aligns with experimental observations of residual hoop stress distribution compared to defining the growth tensor in the natural (zero-stress) configuration [1]. Building upon this work, we propose a new volumetric growth law that departs from the assumption of growth saturation in soft tissues, as this assumption is often invalid in physiologically loaded configurations. Instead, we suggest that growth in soft tissues may lead to a homeostasis state, which does not necessarily imply saturated growth. To quantify this, we introduce a growth incompatibility index (G) and establish an analytical relationship between G and the opening angle (representing residual stress) in a simplified cylindrical geometry resembling the heart or arteries. We present a revised growth law that incorporates this index (G) in place of the limiting function and demonstrate that using this updated growth law, we can accurately depict the timeline of the heart growth, consistent with experimental data obtained from measuring the opening angle in young pigs during weeks 1-10.

References

[1] X. Zhuan. X. Y. Luo, *Volumetric growth of soft tissues evaluated in the current configuration*, Biomechanics and Modeling in Mechanobiology, 21, (2022), 569-588.