On the extended KdV equation, longitudinal bulk strain solitons and undular bores

Karima Khusnutdinova

Department of Mathematical Sciences,
Loughborough University,
Loughborough LE11 3TU, UK

We study long nonlinear longitudinal bulk strain waves in rods and bars within the scope of the general weakly-nonlinear elasticity. We systematically derive the extended Korteweg-de Vries (eKdV) equation and construct a family of approximate weakly-nonlinear soliton solutions with the help of near-identity transformations reducing the eKdV equation to the Gardner equation. These solutions are compared with the results of direct numerical simulations of the full nonlinear problem formulation, showing excellent agreement within the range of their asymptotic validity (waves of small amplitude) and extending their relevance beyond it (to the waves of moderate amplitude, for example, table-top solitons) as a very good initial guess.

Undular bores, or dispersive shock waves, are non-stationary waves propagating as oscillatory transitions between two basic states, in which the oscillatory structure gradually expands and grows in amplitude with distance travelled. Recently, we registered the generation of undular bores in PMMA bars following tensile fracture using high-speed pointwise photoelasticity. We show that a viscoelastic extended Korteweg-de Vries (veKdV) equation provides a very good agreement with the key observed experimental features for a suitable choice of material parameters. Such waves could be present in the signals generated by fracking, earthquakes and other events involving transverse fracture of an appropriately pre-strained waveguide.

References:

[1] F.E. Garbuzov, Y.M. Beltukov, K.R. Khusnutdinova, Longitudinal bulk strain solitons in a hyperelastic rod with quadratic and cubic nonlinearities, Theor. Math. Phys. 202 (2020) 319-333.

[2] C.G. Hooper, P.D. Ruiz, J.M. Huntley, K.R. Khusnutdinova, Undular bores generated by fracture, Phys. Rev. E 104 (2021) 044207.

[3] C.G. Hooper, K.R. Khusnutdinova, J.M. Huntley, P.D. Ruiz, Theoretical estimates of the parameters of longitudinal undular bores in PMMA bars based on their measured initial speeds, Proc. R. Soc. A 478 (2022) 20210867.