Creases and cusps in growing soft matter

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The buckling of a soft elastic sample under growth or swelling has highlighted a new interest in materials science, morphogenesis, and biology or physiology. Indeed, the change of mass or volume is a common fact of any living species, and on a scale larger than the cell size, a macroscopic view can help to explain many features of common observation. Many morphologies of soft materials result from the accumulation of elastic compressive stress due to growth, and thus from the minimization of a nonlinear elastic energy. The similarity between growth and compression of a piece of rubber has revived the instability formalism of nonlinear elastic samples in compression, and in particular the Biot instability. Here we present a modern treatment of this instability in the light of complex analysis and demonstrate the richness of possible profiles that an interface can present under buckling, even if one restricts oneself to the two spatial dimensions. Special attention is given to wrinkles, folds and cusps, a surprising observation in swelling gels or clays. The standard techniques of complex analysis, nonlinear bifurcation theory and path-independent integrals are revisited to highlight the role of physical parameters at the origin of the observed patterns below and above the Biot threshold.