A point of view on quantum field theory

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The following material is not divided into four lectures partly because I'm not sure how much I'll get through, but also because many topics will be mentioned briefly early on and then returned to later in more detail.

Although some of the topics are potentially rather technical, I shall give only a rather general and conceptual introduction to them, not assuming any previous knowledge.

The relation between classical physics and quantum theory, and the ontological role of quantum field theory.

The definition of a quantum system.

The relation between commutative and noncommutative geometry. Connes's concept of a noncommutative manifold. The 'Koszul' viewpoint.

Global aspects of the classical limit of QFTs.

The cobordism-category definition of a QFT. Some examples of its advantages and deficiencies. Gauging and 'higher gauging'.

The space of 2-dimensional QFTs and noncommutative geometry: renormalization group flow as the gradient flow of the Zamolodchikov c-function on the space of theories.

Positivity of energy and its formulation by means of complex metrics using the Dirac operator. Free fermion field theory.

Special features of dimensions 2 and 4 — Ashtekhar's formulation of general relativity.

Consequences of positive energy for the algebra of observables. Two ways of defining field operators.

The consequences of adding a scaling axiom to the definition of QFT.

Tomita-Takesaki theory and the introduction of 'fully extended' theories.

The smooth homotopy category. Deformation quantization. Differential cohomology, and ways of categorifying it as a QFT.