Gravitational bound waveforms from amplitudes

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We develop a formalism to understand the full gravitational two-body dynamics of classical scattering and bound states and their matrix elements using the Schwinger-Dyson equations, with the aim of computing bound observables from amplitudes. Starting with the familiar case of on-shell scattering and bound wavefunctions defined on a Schwarzschild background, we show that they can be analytically continued into each other in the partial wave basis by using a definite branch cut prescription for the incoming energy. The map involves also taking the residue on the bound state pole, which can be avoided by resuming superclassical iterations: this prompts us to study the classical Bethe-Salpeter recursion in the conservative and the radiative case, which can be solved in impact parameter space in terms of an exponential structure connected to two-massive particle irreducible (2MPI) kernels. The relation of these kernels with the Hamilton-Jacobi action and with the waveform is then established. We find that the scattering waveform admits then a natural analytic continuation to the bound waveform at tree-level order, which we explicitly checked by studying the Post-Newtonian expansion of the time-domain multipoles at large angular momentum. Our boundary to bound map agrees also with the Damour-Deruelle prescription for the orbital elements in the quasi-Keplerian parametrization, which enters into the direct evaluation of the time-domain multipoles. Finally, we discuss how our map is consistent with the analytic continuation of the fluxes (i.e, radiated energy and angular momentum) at 3PM order entirely in terms in the binding energy of the system.