

## On Smale's 9th problem, generalised hardness of approximation and the limits of AI

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Alchemists wanted to create gold, Hilbert wanted an algorithm to solve Diophantine equations, researchers want to make deep learning robust in AI without hallucinations, MATLAB wants (but fails) to detect when it provides wrong solutions to linear programs, etc. Why do we fail in so many of these fundamental cases? The reason is typically methodological barriers. The history of science is full of methodological barriers — reasons for why we never succeed in reaching certain goals. In many cases, this is due to barriers in the foundations of mathematics. This talk introduces new such barriers from foundations: the phenomenon of generalised hardness of approximation (GHA). GHA grows out of our work on Smale's 9th problem on the list of mathematical problems for the 21st century. This phenomenon is not a rare issue — but happens on a daily basis in computations (in particular in AI) — and causes modern software such as MATLAB to fail on basic problems, and even certify nonsensical solutions as correct.