Title: Double affine Hecke category and affine Springer fibers

Abstract: I will review conjectures relating coherent realization of the double affine Hecke categories for Langlands dual groups and their action on Springer fibers. I will also report on a joint result in progress with Pablo Boixeda Alvarez, Michael McBreen and Zhiwei Yun which can be thought of as a "single loop" counterpart of a special case of that conjecture.