Speeding up quantum dynamics

From finite to infinite dimensional systems and back

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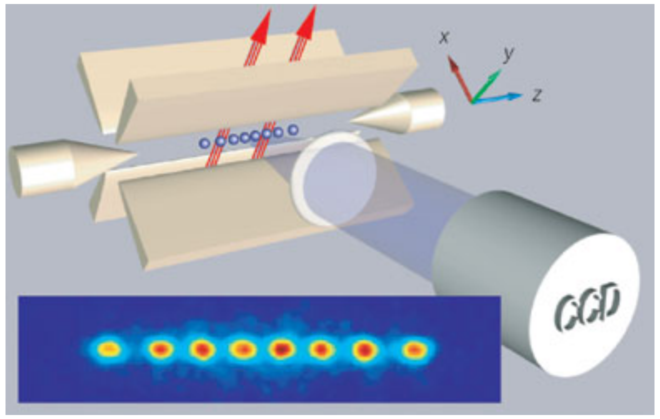
Mathematical physics in quantum technology:
From Finite to Infinite dimensions

Can we do the opposite of dynamical decoupling?

We recall that dynamical decoupling aims to suppress interactions H_0 by the rapid application of controls:

$$M : H_0 \to 0 \qquad (1)$$



Trapped Ions

R. Blatt and D. Wineland, Nature 453, 1008 (2008).

For rapid unitary kicks (bang-bang):

$$M(\cdot) = \frac{1}{|V|} \sum_{v \in V} v^\dagger (\cdot) v$$

For finite dimensional systems, there always exist a decoupling set V such that (1) holds for all traceless H_0.

L. Viola et al., PRL 82, 2417 (1999); C. Arenz et al., JPhys A 50, 135303 (2017)

Does there exist a control strategy, a set V of unitary transformations, that achieves the opposite of (1)?

$$\boxed{M : H_0 \to \lambda H_0, \qquad \lambda > 1 \qquad \text{Hamiltonian Amplification (HA)}}$$

C. Arenz et al., Quantum 4, 271 (2020)

Since for any unitarily invariant norm we have $\|M(H_0)\| \leq \|H_0\|$, HA is impossible (with unitary kicks) for finite dimensional systems.

What about infinite dimensional systems described by unbounded operators?

Outline of the talk

- General control theoretic setting
- Hamiltonian Amplification in infinite dimensions
- Experimental demonstration of Hamiltonian Amplification in an ion trap system
- Speeding up photonic quantum computations
- Conclusions and open questions

Control theoretic setting

D. A. Lidar and T. A. Brun, *Quantum Error Correction* (2013).

Consider a system described by a time dependent Hamiltonian of the form

$$H(t) = H_0 + H_c(t)$$

Possibly unknown Hamiltonian describing the system

Time dependent controller, e.g., shaped electric and magnetic fields

In the frame rotating with the controller the dynamics is given by

$$U(t) = \mathcal{T} \exp\left(-i\int_0^t U_c^\dagger(t')H_0U_c(t')dt'\right) = \exp(-it\bar{H}(t))$$

where $\bar{H}(t)$ is given by the Magnus expansion, which allows for defining HA more generally in a time continuous framework: $\bar{H}(t) = \lambda H_0, \quad \lambda > 1, \text{ for all } t$

There does not exist a controller that achieves HA in finite dimensions

Proof: There always exist a set of unitary transformation $\{W(s)\}$ that allows to express $\bar{H}(t)$ as:

D. A. Lidar et al., Phys. Rev. A 78, 12308 (2008).

$$\bar{H}(t) = \frac{1}{t}\int_0^t W^\dagger(t')U_c^\dagger(t')H_0U_c(t')W(t')dt'$$

For any unitarily invariant norm we then again have $\|\bar{H}(t)\| \leq \|H_0\|$

Hamiltonian Amplification in infinite dimensions

Displacements, interactions and quadratic Hamiltonians

Consider HA described by the rapid application of unitary kicks, so that in the Trotter limit the dynamics is given by

$$\lim_{m\to\infty}\left(\prod_{v\in V_{\mathrm{HA}}} v^\dagger e^{-iH_0\frac{t}{|V|m}} v\right)^m = \exp(-iM_{\mathrm{HA}}(H_0)t), \quad M_{\mathrm{HA}}(H_0) = \frac{1}{|V|}\sum_{v\in V_{\mathrm{HA}}} v^\dagger H_0 v$$

Parametric controls ``squeeze'' the position and momentum operator x, p, described by the unitary transformations $S^{(\pm)}$:

$$S^{(\pm)\dagger} x S^{(\pm)} = x e^{\mp r}, \qquad S^{(\pm)\dagger} p S^{(\pm)} = p e^{\pm r}, \qquad r\in\mathbb{R}$$

- Linear combinations: $H_0 = c_x x + c_p p, \;\; V_{\mathrm{HA}} = \{S^{(\pm)}\}, \;\; M_{\mathrm{HA}}(H_0) = \cosh(r) H_0$

➡ Phase insensitive amplification and enhancement of boson mediated interactions described by
$D(\alpha) = \exp(\alpha^* a + \alpha a^\dagger), \quad H_0 = a^\dagger \otimes B + a \otimes B^\dagger$

- Quadratic Hamiltonians of the form: $H_0 = \sum_{i,j}(\alpha_{i,j} x_i x_j + \beta_{i,j} p_i p_j)$

$$V_{\mathrm{HA}} = \{\mathcal{S}^{(\pm)} = \prod_i S_i^{(\pm)}\}, \qquad M_{\mathrm{HA}}(H_0) = \cosh(2r) H_0$$

➡ Speed up of potentially unknown quantum dynamics

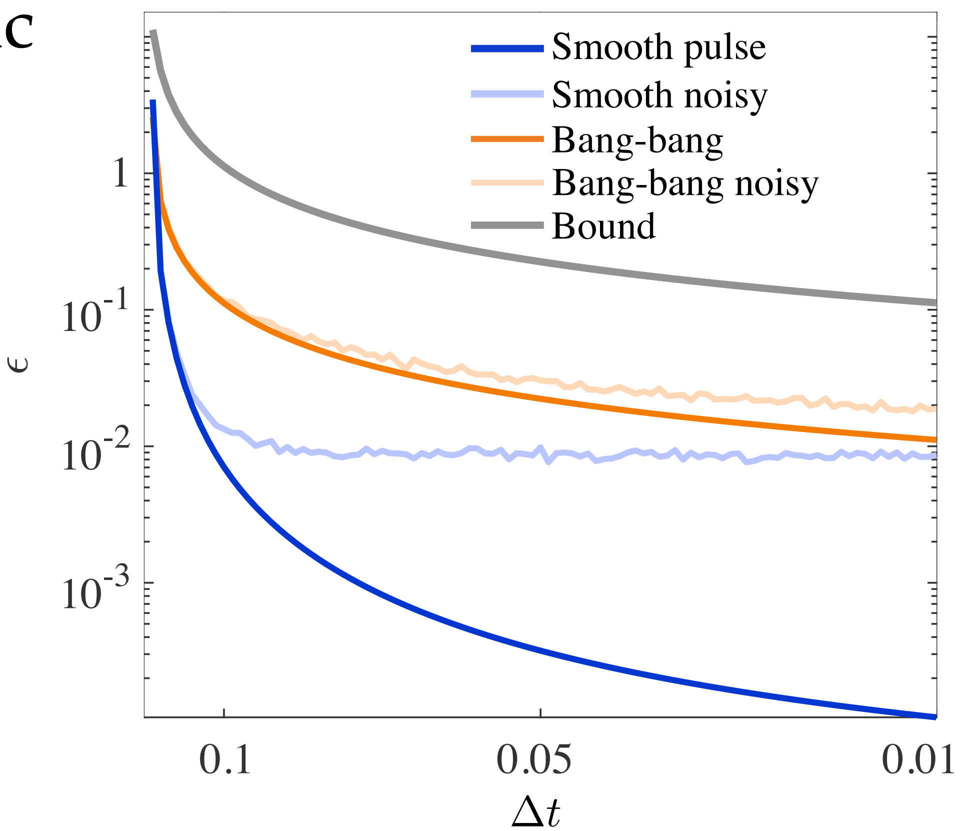
Hamiltonian Amplification in infinite dimensions

Error bounds

Error bounds can be obtained by using the symplectic representation of quadratic Hamiltonians:

$$S = e^{-A_0 \Omega t} \in \mathrm{Sp}(2N, \mathbb{R}), \qquad A_0 = A_0^T \in \mathbb{R}^{2N \times 2N}$$

Contains frequencies and couplings of N harmonic oscillator system



The Trotter error given the Hilbert-Schmidt distance,

$$\epsilon = \left\| e^{-\cosh(2r) A_0 \Omega t} - \left(e^{-\frac{t}{2m} A^{(+)} \Omega} e^{-\frac{t}{2m} A^{(-)} \Omega} \right)^m \right\|,$$

can be upper bounded by

M Suzuki, J. Math. Phys. 26, 601-612 (1985)

$$\epsilon \leq \frac{t \Delta t \omega_{\max}^2 N^2}{2} |\sinh(4r)| \exp\left(t \omega_{\max} N \sqrt{\frac{\cosh(4r)}{2}} \right), \qquad \Delta t = \frac{t}{2m}$$

State dependent Trotter bounds are expected to be tighter

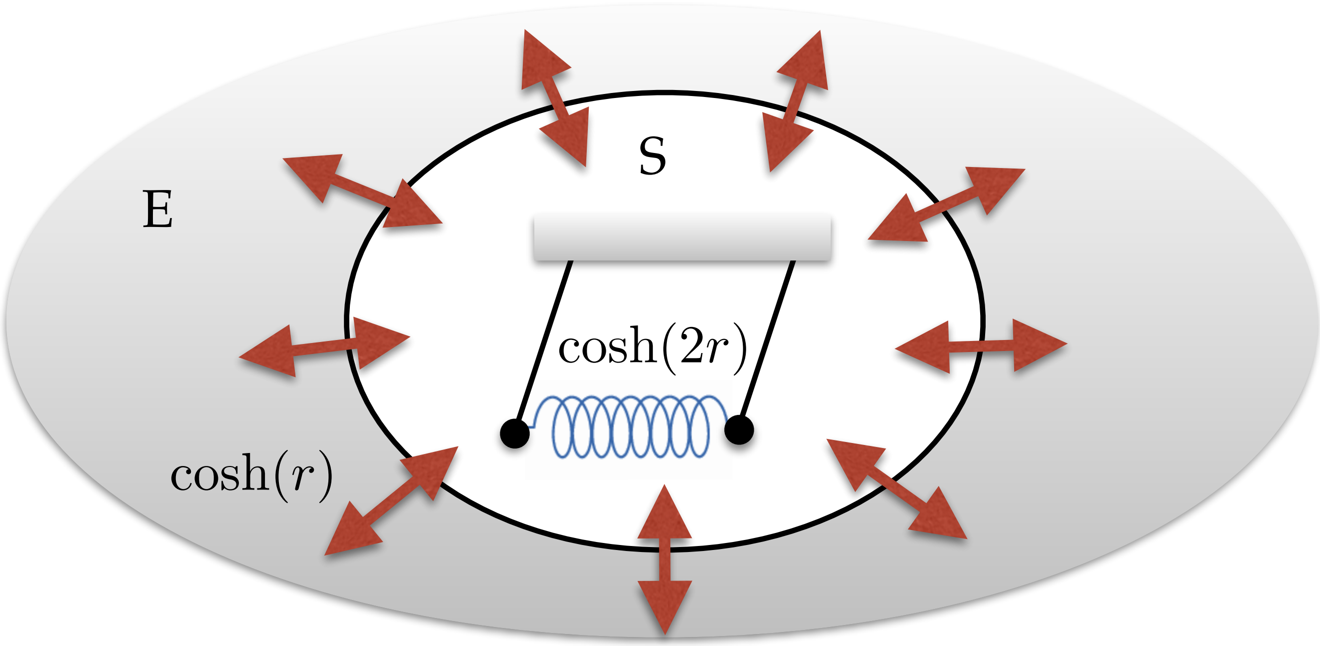
D. Burgarth et al., PRA 107, L040201 (2023) See talk by Alexander Hahn

Hamiltonian Amplification in infinite dimensions

Being faster than decoherence

While quadratic system (S) interactions are amplified through HA, unwanted interactions with the environment (E) that lead to decoherence are amplified too.
If we assume that the interaction with E is described by a quadratic Hamiltonian of the form

$$H_{\mathrm{SE}} = \sum_{i\in S, j\in E} (\alpha_{i,j} x_i x_j + \beta_{i,j} p_i p_j)$$



the total Hamiltonian

$$H_0 = H_{\mathrm{S}} + H_{\mathrm{E}} + H_{\mathrm{SE}}$$

transforms under HA according to :

$$M_{\mathrm{HA}}(H_0) = \cosh(2r) H_{\mathrm{S}} + H_{\mathrm{E}} + \cosh(r) H_{\mathrm{SE}}$$

At times $\tilde{t} = t\cosh(2r)$ decoherence is exponentially $\approx e^{-r}$ suppressed

Selectively enhancing desired interactions while suppressing unwanted interactions at arbitrary times?

Hamiltonian Amplification in infinite dimensions

Dynamical decoupling and Hamiltonian Amplification

Dynamical decoupling of quadratic interactions can be achieved through rapidly rotating the system oscillators describer by,

D. Vitali and P. Tombesi, PRA 59, 4178 (1999)
C. Arenz et al., JPhys A 50, 135303 (2017)

$$\mathcal{R}(\phi) = \prod_{i \in S} e^{-i\phi a_i^\dagger a_i}$$

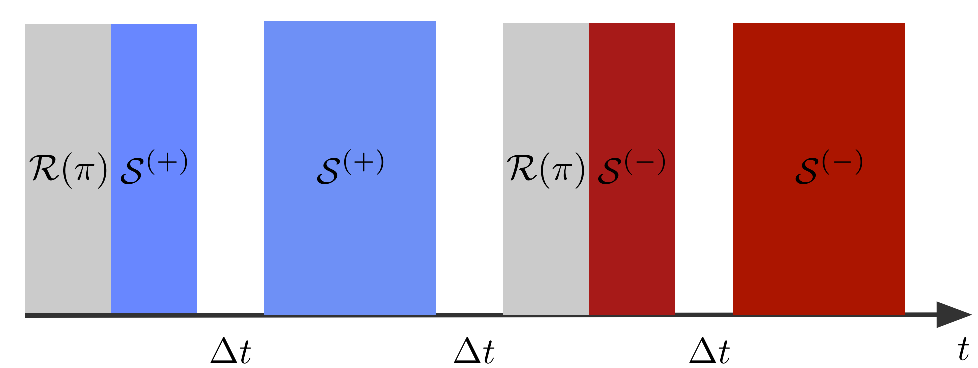
The decoupling set $V_{\mathrm{DC}} = \{\mathcal{R}(0), \mathcal{R}(\pi)\}$ achieves decoupling $M_{\mathrm{DC}}(H_0) = H_{\mathrm{S}} + H_{\mathrm{E}}$

Combining dynamical decoupling and HA,

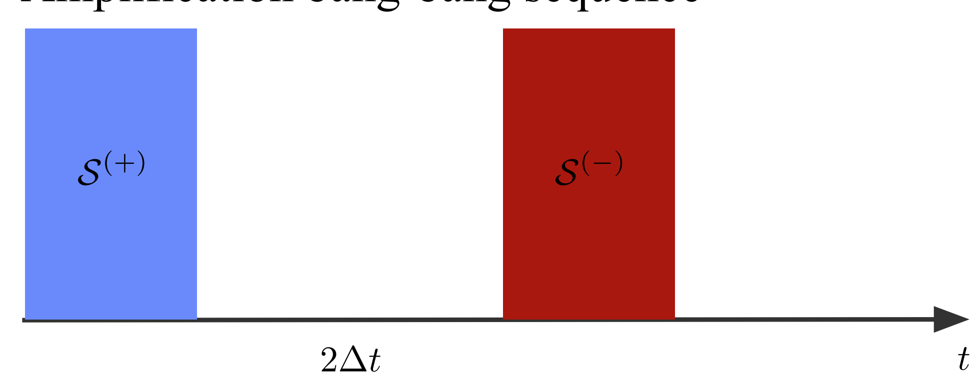
$$M_{\mathrm{HA}}(M_{\mathrm{DC}}(H_0)) = \cosh(2r) H_{\mathrm{S}} + H_{\mathrm{E}},$$

allows for selectively amplifying desired interactions while detrimental interactions are suppressed at all times

Amplification and decoupling bang-bang sequence



Amplification bang-bang sequence



The set of operations simultaneously achieving HA and dynamical decoupling reads

$$\{\mathcal{R}(\pi)\mathcal{S}^{(+)}, \mathcal{S}^{(+)}, \mathcal{R}(\pi)\mathcal{S}^{(-)}, \mathcal{S}^{(-)}\}$$

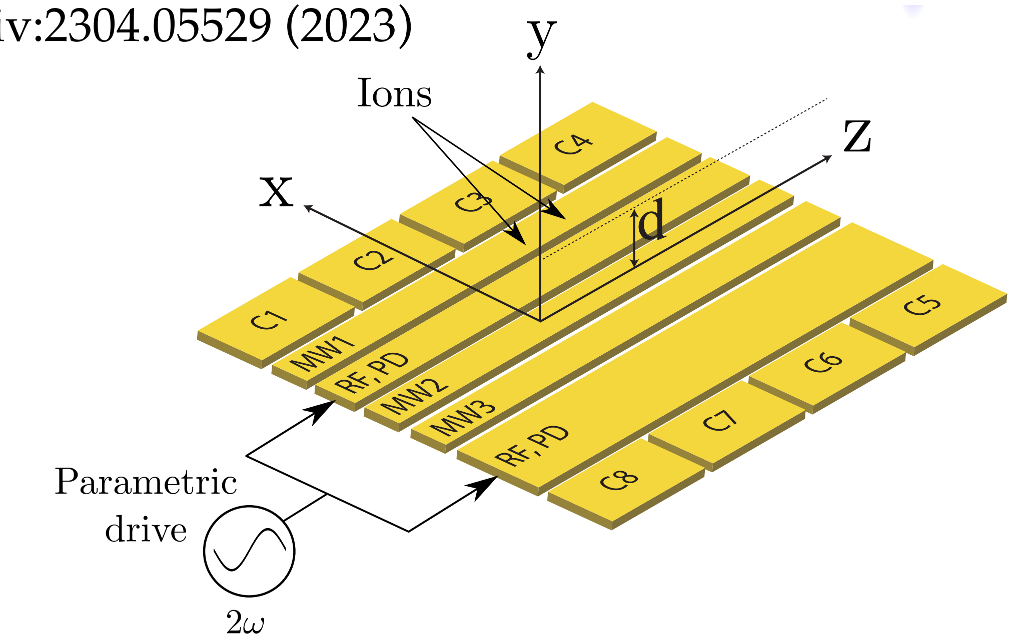
Experimental demonstration of HA

Speeding up Rabi oscillations

S. C. Burd, et al., arXiv:2304.05529 (2023)

Goal: amplify interaction between a quantum harmonic oscillator and a single qubit described by the Jaynes-Cummings interaction:

$$H_0 = a^\dagger \otimes \sigma_- + a \otimes \sigma_+$$

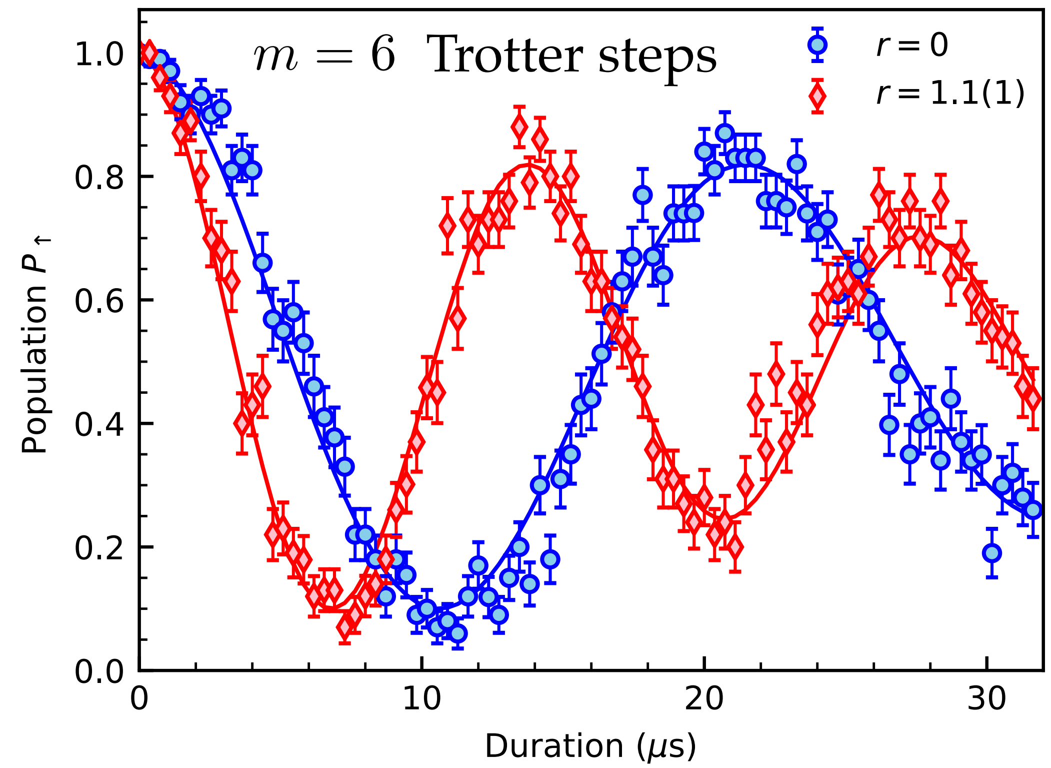


Ion trap setting setting

S. C. Burd et al., Science 364, 6446 (2019)

- Harmonic oscillator: radial motional mode
- Squeezing through oscillating potential at twice the motional frequency
- Qubit: two states of the $^2S_{1/2}$ electronic ground state hyperfine manifold
- Jaynes-Cummings interaction: coupling between qubit and harmonic motion

Rabi oscillations

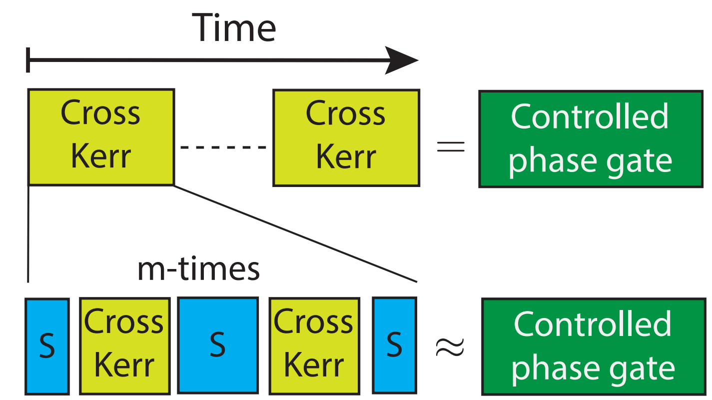


Interaction enhancement of about 3/2

Speeding up photonic quantum computations

Universal quantum computing based on continuous variables requires (higher) non-linear processes.

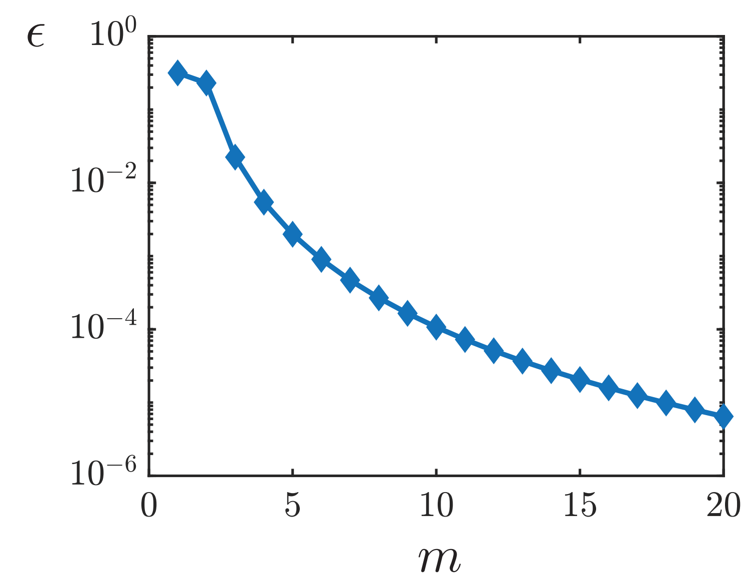
S. Lloyd and S. Braunstein, PRL 82, 1784 (1999)



Universal gate sets in photonic quantum computing can be formed by Gaussian operations, such as beam splitters and phase shifters, and Cross-Kerr interactions:

I.L. Chuang and Y. Yamamoto, PRA 52, 3489 (1995)

$$H_0 = \chi(a^\dagger a \otimes b^\dagger b)$$



Problem: Cross-Kerr interactions are weak compared to the characteristic energy scales of photonic systems.

Use HA to amplify / speed up

Gate error to create a CZ gate: $\epsilon = \left\| \mathrm{CZ} - P\left(S^{(+)\dagger} e^{-iH_0\Delta t} S^{(+)} S^{(-)\dagger} e^{-iH_0\Delta t} S^{(-)}\right)^m P \right\|$

Since HA allows for amplifying the quadratic Hamiltonians that generate beam splitter and phase shifter transformations,

if we assume squeezing is ``free'', quantum algorithms can be implemented arbitrarily fast in photonic systems

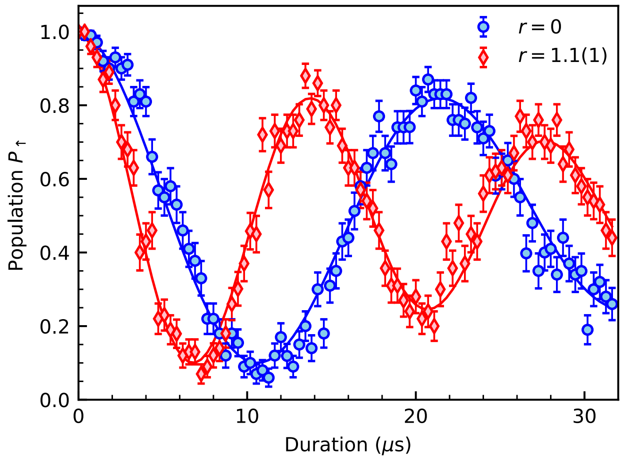
Conclusions and open questions

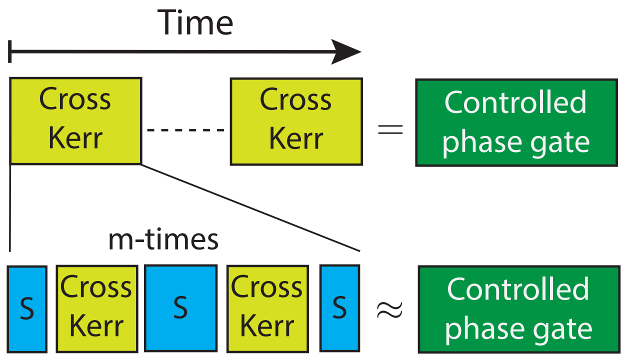
The dynamics of infinite dimensional quantum systems can be sped up by rapidly implementing squeezing along different directions

Dynamical Decoupling

Hamiltonian Amplification

- Enhancing couplings without knowing system details and modifying interactions
- Phase independent amplification of displacement
- Speeding up Rabi oscillations
- Can be combined with dynamical decoupling
- Allows for speeding up photonic quantum computations





Some open questions

What about spin squeezing described by angular momentum operators?

Holstein-Primakoff transformation:

$$\frac{J_-}{\sqrt{2S}} \rightarrow a$$

Angular momentum operators become bosonic operators in the limit of large number S of spins

Upper bound for the Trotter error depending on the number of spins and the number of Trotter steps?

Thank you!

Team

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