Koszulity and Stirling representations

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Stirling numbers of the first (resp. second) kind count permutations (resp. partitions) of {1,2,...,n} with a fixed number of cycles (resp. blocks). Those of the first kind also give the Hilbert function for two graded algebras coming from type A reflection arrangements, or cohomology of configuration spaces: the Orlik-Solmon (OS) algebra and the graded Varchenko-Gelfand (VG) algebra. The representations of the symmetric group on these algebras are very interesting and well-studied.

Since type A reflection arrangements are supersolvable, their OS and VG algebras have quadratic initial ideals and are Koszul algebras, as observed by Peeva (for OS) and by Dorpalen-Barry (for VG). We study here their Koszul dual algebras, whose Hilbert functions are given by Stirling numbers of the second kind. These dual algebras also have quadratic initial ideals, with pleasant standard monomial bases, and carry interesting representations of the symmetric group. We give descriptions of some of these representations, and branching rules, along with some intriguing conjectures.