SECOND ORDER FRACTIONAL MEAN-FIELD SDES WITH SINGULAR KERNELS AND MEASURE INITIAL DATA

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Abstract. In this paper we establish the local and global well-posedness of weak and strong solutions to second order fractional mean-field SDEs with singular/distribution interaction kernels and measure initial value, where the kernel can be Newton or Coulomb potential, Riesz potential, Biot-Savart law, etc. Moreover, we also show the stability, smoothness and the short time singularity and large time decay estimates of the distribution density. Our results reveal a phenomenon that for *nonlinear* mean-field equations, the regularity of the initial distribution could balance the singularity of the kernel. The precise relationship between the singularity of kernels and the regularity of initial values are calculated, which belongs to the subcritical regime in the scaling sense. In particular, our results provide a microscopic probabilistic explanation and establish a unified treatment for many physical models such as the fractional Vlasov-Poisson-Fokker-Planck system, the vorticity formulation of 2D-fractal Navier-Stokes equations, surface quasi-geostrophic models, fractional porous medium equation with viscosity, etc.