# Geometry and the degree-diameter problem

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**Abstract**

The maximum number of vertices in a graph of maximum degree $\Delta \geq 3$ and fixed diameter $k \geq 2$ is upper bounded by $(1 + o(1))(\Delta - 1)^k$. If we restrict our graphs to certain classes of sparse graphs, better upper bounds are known. For instance, for the class of trees there is an upper bound of $(2 + o(1))(\Delta - 1)^{\lfloor k/2 \rfloor}$ for a fixed $k$. I surveyed some of these (not so recent) results, which have been enabled by discrete geometry. Planar graphs, more generally graphs embedded in surfaces of bounded genus, and even more generally $K_r$-minor-free graphs all behave like trees, in the sense that, for large $\Delta$ such graphs have small orders. For instance, there is a constant $c := c(r)$ such that every $K_r$-minor-free graph of diameter $k$ has order at most

$$c(1 + o(1))(\Delta - 1)^{\lfloor k/2 \rfloor};$$

this latter result, due to Bousquet and Thomassé (2015), solved the degree-diameter for such graphs.

Charles Delorme used discrete geometry to construct large graphs of small diameters. In the second part of the talk, I will pose some questions on constructions of graphs of diameters three and four.