Upper bounds for the order of Abelian Cayley graph families in the degree-diameter problem

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Abstract

For given degree and diameter, Abelian Cayley graphs admit a well-known upper bound on their order that is significantly smaller than the Moore bound, often called the *Abelian Cayley Moore bound*. However, we find that the order of every known Abelian Cayley graph is limited by an upper bound that is significantly smaller.

For even degree $d = 2f$, this bound is related to the lowest possible density of a lattice covering of f-dimensional space by orthoplexes (dual hypercubes) centred on the points of a body-centred cubic lattice, with density that is exponential in the dimension. On the other hand, a 1959 theorem by Rogers states that any convex body has a lattice covering with density that is sub-exponential in the dimension, implying the existence of Abelian Cayley graphs that are larger than this assumed upper bound.