Totally regular mixed graphs constructed from $CD(n, q)$ graphs of Lazebnik, Ustimenko and Woldar

Tatiana Jajcayová
Comenius University

Abstract

In 1995, Lazebnik and Ustimenko introduced a family of bipartite q-regular graphs they denoted $D(n, q)$, with q being a prime power and n an integer greater than 1, whose vertex sets consist of two sets comprised of n-dimensional vectors over the finite field $GF(q)$ with the adjacency between pairs of vertices from the two sets determined via a series of identities to be satisfied by the coordinates of the two vectors. Shortly afterwards, the original authors together with Woldar have shown the graphs to be disconnected, and denoted their connected components by $CD(n, q)$. At this point, the $CD(n, q)$ graphs constitute the best universal family of regular graphs of prime power degree with regard to the *Cage Problem*. Their girths are known to be at least $n + 4$ in case of even n, and $n + 5$ for odd n.

We propose to extend the use of the $CD(n, q)$ graphs into the area of *mixed graphs* by adding direction to certain edges of the $CD(n, q)$ graphs. In the case when $q = 3$, our construction yields graphs having the property that each vertex is adjacent to one (non-oriented) edge, one out-going, and one in-going edge. In the context of mixed graphs, graphs in which the numbers of non-oriented, out-going and in-going edges are equal and the same for all vertices are of special interest and have been named *totally regular mixed graphs*.

In view of the special properties of the original graphs $CD(n, q)$ with regard to cages, we believe that the totally regular mixed graphs we propose to study may also prove to be extremal with regard to properties sought for in the area of mixed graphs. We will also consider generalizations of our constructions to the case of degree which is a power of 3.