Upper bounds on the orders of cages

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Abstract

The Cage Problem is the problem of finding a k-regular graph of girth g of the smallest possible order, a (k, g)-cage, for any given pair k, g. It is universally acknowledged to be a hard problem which is fully solved only for an extremely limited set of (small or special) parameter pairs k, g. While the existence of any k-regular graph of girth g provides us with an upper bound on the order of the corresponding (k, g)-cages, the orders of even the smallest known such graphs are in general significantly bigger than the best available lower bounds.

In our presentation, we present a brief survey of the best known upper bounds on the orders of (k, g)-cages and the corresponding graph constructions. We also outline some immediate and long term challenges in this area.