

Lipschitz images of the Cantor set

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The Analyst's Traveling Salesman Problem is to characterize those sets that can be covered by a Lipschitz image of $[0,1]$. We study the problem we get by replacing the interval by the Cantor set. Another motivation comes from the well known classical result that the compact metric spaces are exactly the continuous images of the Cantor set, so it seems to be natural to ask which metric spaces can be obtained as a Lipschitz image of the Cantor set. We prove that every compact metric space of upper box dimension less than $\log 2 / \log 3$ can be obtained as the Lipschitz image of the Cantor set. We characterize those self-similar sets with the strong separation condition that can be obtained as the Lipschitz image of the Cantor set. In fact, we prove more general results than these ones and we also have other results that we needed or obtained as a spin off. Among others we show that in some sense every reasonable fractal dimension must be at least the Hausdorff dimension and at most the upper box dimension and we give a characterization of those compact metric spaces that can be obtained as an α -Hölder image of $[0,1]$. This is joint work with Richárd Balka.