Title: Regularity properties of parallel volume and parallel surface area

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Abstract: Given a compact set A in R^d, the r-parallel sets A_r are a particularly nice way to approximate A (as the parallel radius r tends to 0), encoding much of the geometry of A. They are the key to many geometric quantities such as Minkowski contents, curvatures measures, and geometric zeta functions. It is well known that the volume function of A (associating to r the volume of A_r) is differentiable at all r>0 except countably many and that its derivative is related to the surface area of A_r. We discuss localizations of this result and some consequences, e.g. the weak convergence of the surface area measures of r-parallel sets of A to the surface area measure of the s-parallel set as r → s, provided s is a differentiability point of the volume function.

We also address the question which (countable) sets of parallel radii are possible as sets of non-differentiability points of the volume function of some compact set. We provide a full characterization for dimensions 1 and 2.

Based on joint work with Jan Rataj.