Proving a ratio limit theorem for random walks on groups using the variational principle.

Rhiannon Dougall
Durham University

(This talk will be close the topic of group extensions.) There are strong results for certain random walks on groups when we have enough structure, such as the random walk being symmetric and the group having some good structure. We can manage without symmetry if the group is small enough e.g. abelian.
We will be interested in the case of a non-degenerate (non-symmetric in general) random walk on an amenable group. We are able to obtain a ratio limit theorem, which says that the probability to return to g in n steps is asymptotically proportional to that of returning to some fixed origin; and we have an explicit description of the constant in terms of g.
We'll describe how this result follows from large deviations and the variational principle.
This is joint work with Richard Sharp.

Dimensions of arithmetic sums of typical self-affine sets

De-Jun Feng
The Chinese University of Hong Kong

An important problem in fractal geometry is understanding fractal dimensions of self-affine sets.

In 1988, Falconer introduced the concept of affinity dimension, and showed that the Hausdorff and box-counting dimensions of a typical self-affine set (in some sense) are equal to its affinity dimension. In this talk, I will present some results on the dimensions of arithmetic sums of typical self-affine sets. It is based on joint work with Yu-Hao Xie.

A dichotomy on the self-similarity of graph-directed attractors

Jiaxin Hu
Tsinghua University

We seeks conditions that ensure that the attractor of a graph directed iterated function system (GD-IFS) cannot be realised as the attractor of a standard iterated function system (IFS). For a strongly connected directed graph, it is known that, if all directed circuits go through a vertex, then for any GD-IFS of similarities on $\mathbb{R}$ based on the graph and satisfying the convex open set condition (COSC), its attractor associated with this vertex is also the attractor of a (COSC) standard IFS. We show the following complementary result. If a directed circuit does not go through a vertex, then there exists a GD-IFS based on the graph such that the attractor associated with this vertex is not the attractor of any standard IFS of similarities. Indeed, we give algebraic conditions for such GD-IFS attractors not to be attractors of standard IFSs, and thus show that `almost-all' COSC GD-IFSs based on the graph have attractors associated with this vertex that are not the attractors of any COSC standard IFS. This is a recent joint work with Kenneth Falconer and Junda Zhang.

Maarit Järvenpää
University of Oulu

Title: The shrinking target problem for certain self-affine sets.

Thomas Jordan
University of Bristol

Abstract: For a dynamical system you can consider the starting points of orbits which hit a set of shrinking targets infinitely often. This set is usually known as the shrinking target set. You can study whether the set has zero or full measure for some suitable invariant measure and in appropriate setting you can consider the Hausdorff dimension of the set. For conformal systems in $\R^d$ there problems are fairly well understood but as usual much less is known in the non-conformal setting. We consider a special class of self-affine sets on the plane and make use of both transversality and recent work by Shmerkin on the L6q spectra of self-similar measures to find the dimension of the shrinking target sets. We will also highlight differences with the self-similar and conformal cases. This is joint work with Henna Koivusalo.

Hausdorff dimension of the Rauzy gasket

Natalia Jurga
University of St Andrews

The Rauzy gasket is a fractal subset of the two dimensional simplex which is an important subset of parameter space in numerous dynamical and topological problems. Arnoux conjectured that the Hausdorff dimension of the Rauzy gasket is strictly less than 2, and since then there has been considerable interest in computing its Hausdorff dimension.

In this talk we will also discuss the Furstenberg measures for the action of the generators of the Rauzy gasket on the projective plane. We will discuss how recent developments in the theory of self-affine measures can be adapted to compute the dimension of these Furstenberg measures, which will allow us to establish an exact value for the Hausdorff dimension of the Rauzy gasket.

Lipschitz images of the Cantor set

Tamás Keleti
Eötvös Loránd University, Budapest

The Analyst's Traveling Salesman Problem is to characterize those sets that can be covered by a Lipschitz image of [0,1]. We study the problem we get by replacing the interval by the Cantor set. Another motivation comes from the well known classical result that the compact metric spaces are exactly the continuous images of the Cantor set, so it seems to be natural to ask which metric spaces can be obtained as a Lipschitz image of the Cantor set. We prove that every compact metric space of upper box dimension less than log2/log3 can be obtained as the Lipschitz image of the Cantor set. We characterize those self-similar sets with the strong separation condition that can be obtained as the Lipschitz image of the Cantor set. In fact, we prove more general results than these ones and we also have other results that we needed or obtained as a spin off. Among others we show that in some sense every reasonable fractal dimension must be at least the Hausdorff dimension and at most the upper box dimension and we give a characterization of those compact metric spaces that can be obtained as an α-Hölder image of [0,1]. This is joint work with Richárd Balka.

Dimension of \tau-badly approximable points as a subset of \tau-well approximable points

Henna Koivusalo
University of Bristol

The first mass transference principle (MTP) was proved by Beresnevich and Velani in 2005. Roughly speaking, MTP says that if the limsup set given by a sequence of balls is large (full measure), shrinking the balls in a controlled way does not reduce the size of the limsup set too much (the dimension can be bound from below). MTP and its variants are an excellent tool for finding lower bounds for limsup-type sets. It has found many applications, including in Diophantine approximation, where it has been used to study well approximable points: Those points which lie in the limsup set of neighbourhoods of rational points.

We provide a new technique which relies on MTP but allows for the study of a particular liminf set: the set of badly approximable points. These are the points which eventually lie outside of the neighbourhoods of rationals. In particular, we show that the set of \tau-badly approximable points as a subset of \tau-well approximable points has full dimension. The special case of ambient dimension 1 was presviously studied by Bugeaud, but his proof technique relies on continued fractions and does not generalise to higher dimensions.

The talk is based on a joint work with Ben Ward, Jason Levesley and Xintian Zhang.

Hausdorff dimension and principal values of singular integrals

Pertti Mattila
University of Helsinki

This is a joint work with Julia Cufi, Juan Jesus Donaire and Joan Verdera. We show that on some standard Cantor sets in the plane with Hausdorff dimension 1 and non-sigma-finite one-dimensional Hausdorff measure, the principal values of the Cauchy transform with respect to the natural probability measure exist on a set of Hausdorff dimension one, although they often fail to exist almost everywhere. I shall also discuss questions related to more general settings.

Multifractal zeta-functions via Fredholm determinants

Lars Olsen
University of St Andrews

For self-conformal graph-directed iterated function systems (SGIFS) we de ne multifractal zeta-functions using Fredholm determinants.

We show that there is an intimate connection between (the analytic continuation of) the zeta-functions and the multifractal dimensions of the self-conformal measures associated with the SGIFS. In particular, the abscissa of convergence equals the multifractal dimension and the poles provides detailed information about the rate of convergence of the multifractal dimensions.

We also obtain exact growth estimates of the multifractal zeta-functions and their eigenvalues significantly improving previous estimates by, for example, Bandtlow, Jenkinson and Pollicott.

Several examples are considered. For example, if the SGIFS consists of similarities in R, then the multifractal zeta-function is a re ned version the Lapidus geometric zeta-function, and if the SGIFS generates the limit set of a Schottky group, then the multifractal zeta-function is a weighted Selberg zeta-function.

Some recent developments on the Falconer distance set problem

Pablo Shmerkin
University of British Columbia

I will survey some recent developments around the celebrated Falconer distance set problem, in particular focusing on the dimension of the sets of distances spanned by sets of critical dimension. Based on joint work with Hong Wang (part of which is in turn based on joint work also with Tuomas Orponen).

Metric spaces in random geometry: dimension theory and applications

Sascha Troscheit
University of Oulu

In this talk I will give an overview of how methods from dimension theory can be used to study regularity properties of random metric spaces. In particular, I will talk about the Assouad dimension and its role in determining embeddability of classic random spaces: the Brownian Continuum Random Tree and the Brownian Map. I will also talk about how regularity properties such as dimensions change when deterministic "fractal" objects in R change in random geometries.

Title: Regularity properties of parallel volume and parallel surface area

Steffen Winter
Karlsruhe Institute of Technology

Abstract: Given a compact set A in R^d, the r-parallel sets A_r are a particularly nice way to approximate A (as the parallel radius r tends to 0), encoding much of the geometry of A. They are the key to many geometric quantities such as Minkowski contents, curvatures measures, and geometric zeta functions. It is well known that the volume function of A (associating to r the volume of A_r) is differentiable at all r>0 except countably many and that its derivative is related to the surface area of A_r. We discuss localizations of this result and some consequences, e.g. the weak convergence of the surface area measures of r-parallel sets of A to the surface area measure of the s-parallel set as r → s, provided s is a differentiability point of the volume function.

We also address the question which (countable) sets of parallel radii are possible as sets of non-differentiability points of the volume function of some compact set. We provide a full characterization for dimensions 1 and 2.

Based on joint work with Jan Rataj.

Thickness and a Gap Lemma in $\mathbb{R}^d$

Alexia Yavicoli
The University of British Columbia

A general problem that comes up repeatedly in geometric measure theory, dynamics and analysis is understanding when two or more "small" compact sets intersect. In the real line, the classical Gap Lemma of S. Newhouse, based on the notion of thickness, gives an easily checkable robust condition for two Cantor sets to intersect, but it is strongly based on the order structure of the reals. I will discuss some recent extensions of the notion of thickness, and the Gap Lemma, to higher dimensions. Applications to patterns in fractals will be discussed.

Arithmetic properties of missing-digit numbers

Han Yu
University of Warwick

For an integer $b>2$, consider the set C of real numbers whose base b expansions have only digits in a certain set of digits. This C is a nice fractal set and is well-studied in Geometric Measure Theory. In this talk, we will explore some problems concerning some arithmetic properties of C, e.g. the sumset $C+C$, quotient set C/C, and product set $C*C$. In addition, we will also discuss some recent results on Diophantine Approximation on C. Part of this talk is based on joint work with Demi Allen (Exeter) Sam Chow (Warwick) and Peter Varju (Cambridge).