A dichotomy on the self-similarity of graph-directed attractors

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We seeks conditions that ensure that the attractor of a graph directed iterated function system (GD-IFS) cannot be realised as the attractor of a standard iterated function system (IFS). For a strongly connected directed graph, it is known that, if all directed circuits go through a vertex, then for any GD-IFS of similarities on $\mathbb{R}$ based on the graph and satisfying the convex open set condition (COSC), its attractor associated with this vertex is also the attractor of a (COSC) standard IFS. We show the following complementary result. If a directed circuit does not go through a vertex, then there exists a GD-IFS based on the graph such that the attractor associated with this vertex is not the attractor of any standard IFS of similarities. Indeed, we give algebraic conditions for such GD-IFS attractors not to be attractors of standard IFSs, and thus show that `almost-all' COSC GD-IFSs based on the graph have attractors associated with this vertex that are not the attractors of any COSC standard IFS. This is a recent joint work with Kenneth Falconer and Junda Zhang.