Dimension of \tau-badly approximable points as a subset of \tau-well approximable points

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The first mass transference principle (MTP) was proved by Beresnevich and Velani in 2005. Roughly speaking, MTP says that if the limsup set given by a sequence of balls is large (full measure), shrinking the balls in a controlled way does not reduce the size of the limsup set too much (the dimension can be bound from below). MTP and its variants are an excellent tool for finding lower bounds for limsup-type sets. It has found many applications, including in Diophantine approximation, where it has been used to study well approximable points: Those points which lie in the limsup set of neighbourhoods of rational points.

We provide a new technique which relies on MTP but allows for the study of a particular liminf set: the set of badly approximable points. These are the points which eventually lie outside of the neighbourhoods of rationals. In particular, we show that the set of \tau-badly approximable points as a subset of \tau-well approximable points has full dimension. The special case of ambient dimension 1 was presviously studied by Bugeaud, but his proof technique relies on continued fractions and does not generalise to higher dimensions.

The talk is based on a joint work with Ben Ward, Jason Levesley and Xintian Zhang.