The Poincare-Extended ab-Index

Galen Dorpalen-Barry

Ruhr-University Bochum

Motivated by a conjecture of Maglione—Voll concerning Igusa zeta functions, we introduce and study the Poincaré-extended ab-index. This polynomial generalizes both the ab-index and the Poincaré polynomial. For posets admitting R-labelings, we prove that the coefficients are nonnegative and give a combinatorial description of the coefficients. This proves Maglione—Voll's conjecture as well as a conjecture of the Kühne—Maglione. We also recover, generalize, and unify results from Billera—Ehrenborg—Readdy, Ehrenborg, and Saliola—Thomas.