Robust and adaptive estimation of a density on the line under a shape constraint

Yannick Baraud

University of Luxembourg

Abstract:

Using the L1-loss as a measure of accuracy, we propose to estimate a density on the line under the assumption that it satisfies one of the following shape constraints on its support: monotonicity, convexity, concavity or log-concavity. To do so, we have at disposal n data, that are presumed to form a n-sample of the target density, and we show that it is possible to design an estimator which possesses the following properties. Firstly, the estimator is only based on the data and the presumed shape of the density. In particular, it does not require to know the support of this density. Then, the estimator is able to reach parametric rates of convergence when the target density possesses some specific features. This property is usually called adaptation in the literature. Finally, the estimator still performs well when the data are not i.i.d. but only independent and most of the marginal densities are close enough to a density which do satisfy the desired shape constraint. This property accounts for the robustness properties of the estimator, i.e. robustness with respect to both the equidistribution assumption and a possible misspecification of the shape. Finally, we shall see how these results easily derive from the combination of a non-asymptotic risk bound on the estimator, involving explicit constants, and some new results in approximation theory.

This work has been done in collaboration with Hélène Halconruy and Guillaume Maillard.