Confidence bands for distribution functions: a new approach



Jon A. Wellner

University of Washington, Seattle

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Abstract

We introduce new goodness-of-fit tests and new confidence bands for distribution functions motivated by multi-scale methods of testing and based on laws of the iterated logarithm for the the normalized uniform empirical process $\mathbb{U}_n(t)/\sqrt{t(1-t)}$ and its natural limiting process $\mathbb{U}(t)/\sqrt{t(1-t)}$. The new goodnessof-fit tests and confidence bands refine the procedures of Berk and Jones and Owen (1979) and Owen (1995). Roughly speaking, the high power and accuracy of the latter procedures in the tail regions of distributions are essentially preserved while gaining considerably in the the central region. The goodness of fit tests perform well in signal detection problem involving sparsity as in Donoho and Jin (2004) and Jager and Wellner (2007), but also under contiguous alternatives. Our analysis of the confidence bands sheds new light on the influence of the underlying ϕ -divergences.