Second- and Higher-Order Anti-Concentration Inequalities, Comparison Theorems and Bootstrap

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Abstract:

We present some second- and higher-order Gaussian anti-concentration inequalities in high-dimension and error bounds in Slepian's comparison theorem for the distribution functions of the maxima of two Gaussian vectors. The anti-concentration theorems are presented as upper bounds for the sum of the absolute values of the partial derivatives of a certain order for the joint distribution function of a Gaussian vector or weighted sums of such absolute values. The second order anti-concentration inequality is used to develop comparison theorems for the joint distribution functions of Gaussian vectors or equivalently the univariate distribution functions of their maxima via Slepian's interpolation. The third and higher order anti-concentration inequalities are motived by recent advances in the central limit theorem and consistency of bootstrap for the maximum component of a sum of independent random vectors in high-dimension and related applications in statistical inference and machine learning.