Cell biology is a fascinating and complex area of research with a myriad of unanswered questions. Many important biological processes involve large changes in cell morphology as well as cell movement such as wound healing, embryonic development and cancer metastasis. Mathematical modelling is being increasing used to shed light on the detailed interplay of extracellular and intracellular signalling events which are often mediated by the cell plasma membrane. The mathematical and computational challenges are that these problems are often posed on time evolving domains and take the form of bulk-surface PDEs. I will present some developments on the use of adaptive moving mesh methods to tackle these problems. These new methods will be applied to a forced curve shortening flow model to describe an evolving cell membrane as well as the image segmentation of an evolving cell from experimental video data. The flexibility of these moving mesh methods will also be demonstrated when applied to the simulation of single cell chemotaxis which is driven by external environmental signals.