This is joint work with Chris Budd (Bath), Jemma Shipton (Exeter), and Colin Cotter (Imperial). In this talk, I will discuss the generation of meshes adapted to a prescribed scalar 'monitor' function through equidistribution. When supplemented with an optimal transport condition, in the planar case the resulting mesh can be obtained by solving a Monge-Ampère equation, a scalar nonlinear elliptic PDE. This approach can be generalised to other manifolds such as the sphere, which leads to a similar equivalent equation, modified for the spherical geometry. In the second part of this talk, I discuss the integration of moving mesh adaptivity into a finite element shallow water model on the sphere. We do this by modifying the governing fluid equations so they are solved in a frame relative to the moving mesh. The finite element discretization is based on a 'compatible', or 'mimetic', approach, in which the finite element spaces are linked by differential operators. The degrees of freedom correspond not just to point values, but also to fluxes and densities, which complicates the modifications that are required.