

Title: Maximal averages along hypersurfaces: a "geometric conjecture" and further progress for 2-surfaces

Abstract: The study of L_p -estimates for maximal averages along hypersurfaces originated in E.M. Stein's seminal work on dimension free estimates for the Hardy-Littlewood maximal operator, in which he had studied the spherical maximal function. In this talk, I shall first outline what has been known so far for more general smooth, finite type 2 - hypersurfaces, for which the sharp range of p s such that the associated maximal operator is L_p - bounded can be described in terms on Newton polyhedra, and then explain more recent progress obtained in joint work with S. Buschenhenke, S. Dendrinos and I. Ikromov.

This work has also led us to a new "geometric conjecture" on maximal averages along hypersurfaces in arbitrary dimension, which we can show to hold true for all analytic 2-surfaces, with the exception of a small class of hypersurfaces with singularities of type A, whose study is left open. It has become clear from our studies of examples that a proof of our geometric conjecture for surfaces from this exceptional class will require some of the deepest techniques of modern harmonic available, as there are close connections to the reverse square function estimate for the cone multiplier, which has recently been proved by L. Guth, H. Wang and R. Zhang.