Let S be a del Pezzo surface of degree one over an infinite field. After blowing up a specific point one obtains an elliptic surface E. Several results on the density of the set of rational points S(k) in S require for S(k) to contain a point which is non-torsion on its fiber on E, and/or not contained in `many' exceptional curves on S. I will talk about the relation between these two conditions: is every point on S that is contained in `many' exceptional curves torsion on its fiber on E?\\
After motivating this question in more detail, I will show that the answer is yes if `many' equals at least 9, and no if `many' equals 7 (which is joint with Julie Desjardins). The question for 8 exceptional curves is still work in progress.