Let $f = a_1 x_1^3 + \dots + a_n x_n^3$ be a diagonal cubic polynomial with coefficients a_i in $F_q(t)$ and let $N(P)$ be the number of $F_q[t]$-solutions to $f = 0$ where each variable has degree at most P. Using the delta method we show that when $n=6$ we obtain an upper bound of $O(|P|^{3+\varepsilon})$ for $N(P)$ and when $n=4$ we obtain an upper bound of $O(|P|^{3/2+\varepsilon})$ for $N(P)$ away from rational lines on the cubic hypersurface. This is the $F_q(t)$ analogue of work by Heath-Brown (1998), who obtains the same results in the rational numbers setting, albeit conditionally on some hypotheses on certain Hasse-Weil L-functions. Going back to our setting, these hypotheses can be shown to hold using Deligne's proof of the Weil conjectures, which has been carried out by Browning and Vishe (2015).

As a consequence we establish the asymptotic version of Waring's problem over $F_q(t)$ in 7 variables and weak approximation for diagonal cubic hypersurfaces in at least 7 variables.

This is joint work with Jakob Glas.