If a Kummer surface has good reduction, then it comes from an abelian surface with good reduction. The converse holds if the residual characteristic is not 2. In a joint work with Chris Lazda we obtain a necessary and sufficient condition for good reduction of Kummer surfaces attached to abelian surfaces with good, non-supersingular reduction in residual characteristic 2. As an arithmetic application, on such surfaces it is easy to construct Brauer classes of order 2 with Swan conductor zero, hence with constant evaluation map for any finite field extension.