Stabilized Finite Element Multigrid Techniques for Space-Time Parallelism in Convection-Diffusion Problems

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Convection-dominated problems pose significant challenges for the numerical solution of time-dependent PDEs, particularly in the limit of low or vanishing diffusion. In such regimes, standard Galerkin discretizations often produce non-physical oscillations, which are typically mitigated by stabilization techniques. However, there is a need for solution strategies that respect the underlying physics and perform robustly across the full spectrum from parabolic to hyperbolic behavior, particularly in the context of PinT methods.

In this talk, we explore a space-time parallelization framework that combines a time-simultaneous approach, closely related to multigrid waveform relaxation, with space-time multigrid solvers. This enables the simultaneous solution for multiple time steps while maintaining spatial parallelism, providing a natural framework for exploiting massive concurrency on emerging exascale architectures.

We investigate this methodology using the one-dimensional convection-diffusion equation as a model problem. A key ingredient is the use of variational multiscale stabilization, which improves both discretization accuracy and solver performance in convection-dominated regimes. Emphasis is placed on the interaction between discretization and solver design, and we discuss how the proposed multigrid approach yields robust performance across a wide range of diffusion coefficients.