Walter Strauss

Modulational Instability of Stokes Waves

I will present an exposition of joint work with Huy Quang Nguyen. We prove rigorously that the classical (small-amplitude irrotational steady periodic) water waves are unstable with respect to long-wave perturbations. This instability was first observed heuristically more than half a century ago by Benjamin and Feir. A rigorous proof was found by Bridges and Mielke in 1995, but only in the case of finite depth. We provide a completely different and self-contained proof of both the finite and infinite depth cases. The proof reduces to an analysis of the spectrum of an explicit operator. The growth is obtained by means of a rather subtle Liapunov-Schmidt reduction that more or less reduces the analysis to four dimensions.