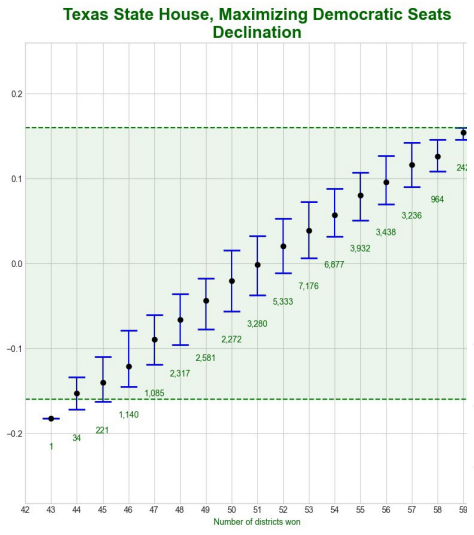
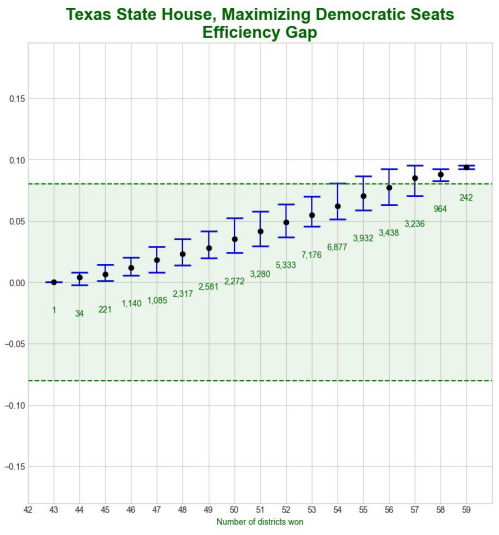
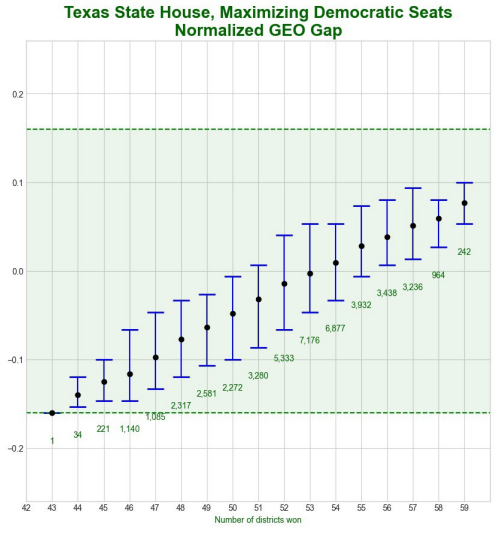
Analyzing metrics to detect gerrymandering via short bursts

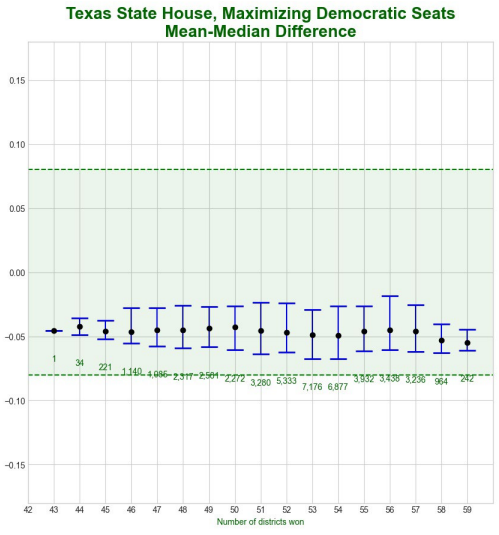
Ellen Veomett

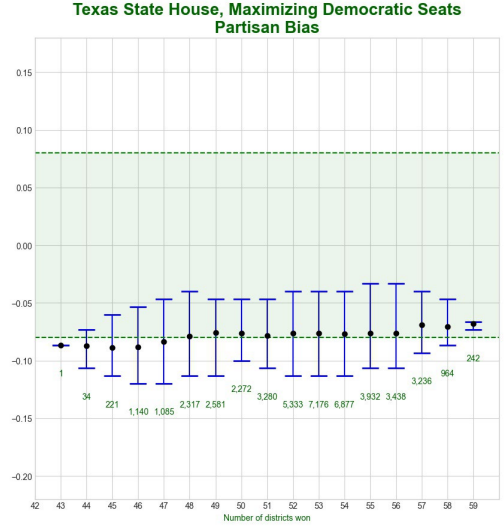
University of San Francisco











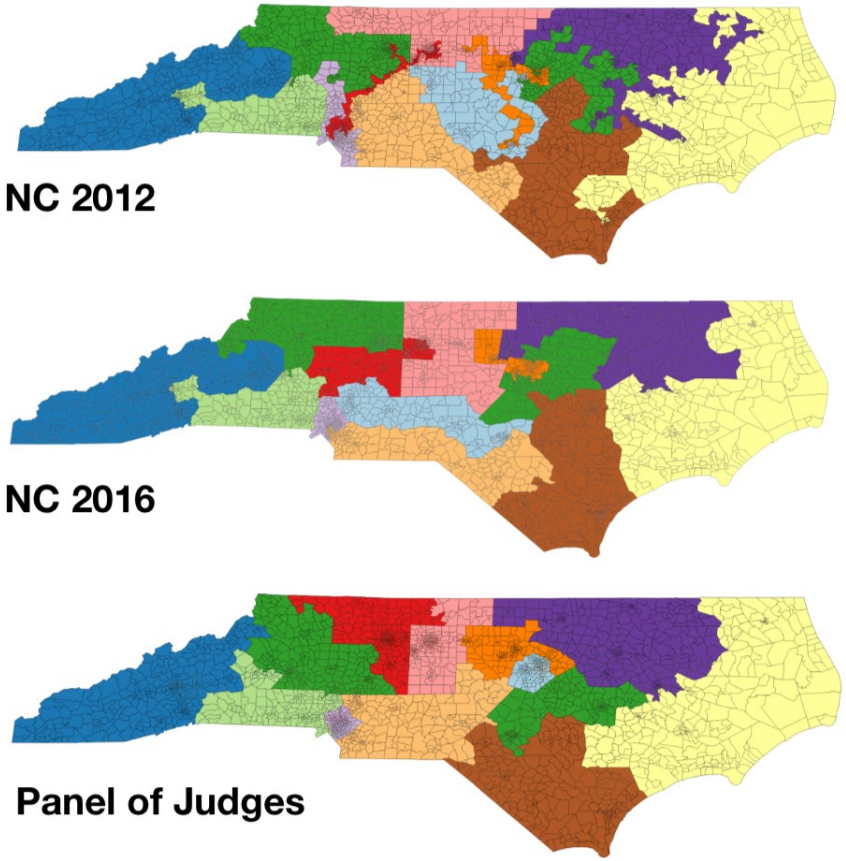
Goals of this Talk

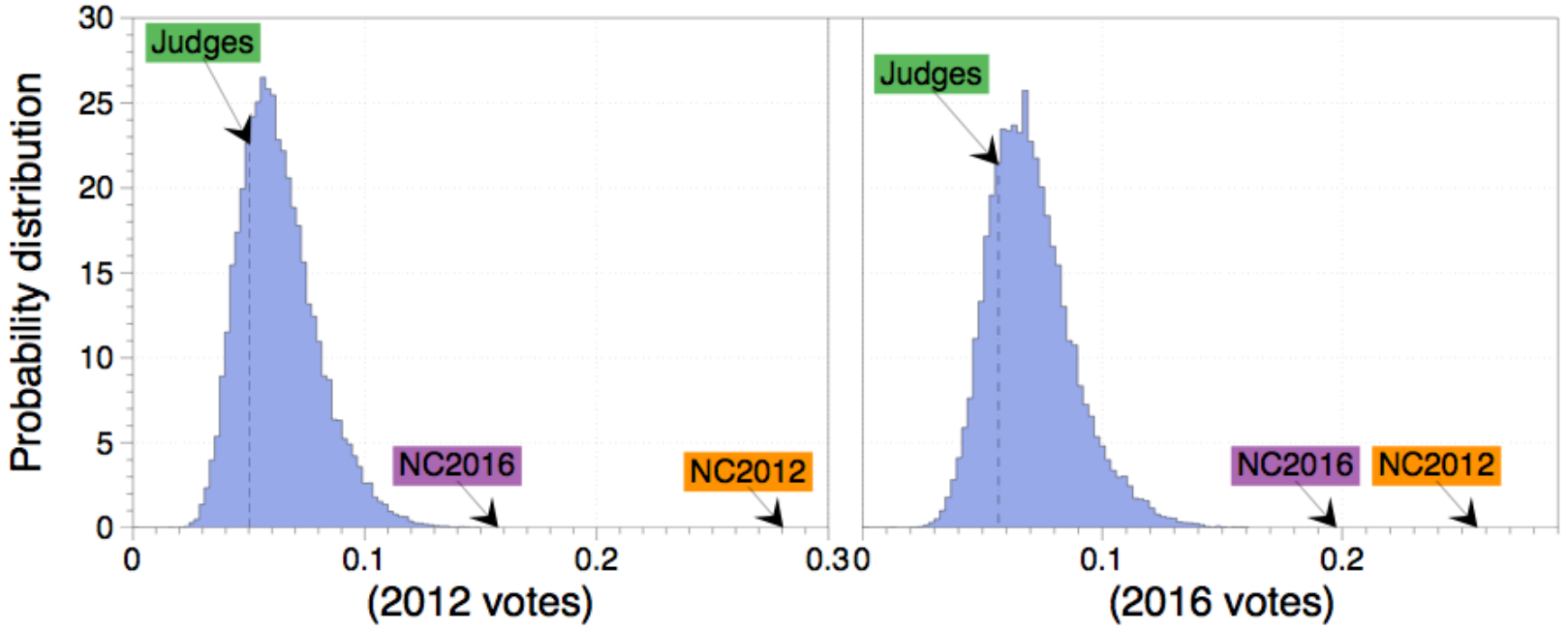
- Introduce you to metrics to detect gerrymandering
- Share theoretical results explaining how those metrics *can* work
- Share empirical results showing whether those metrics *do* work
  - If time: give ideas for why
- Convince you that the Mean Median Difference and Partisan Bias should only be used with extreme caution
  - Should not be used at all?

How Can We Detect Gerrymandering?

Outlier analysis: Compare *proposed map* to *neutral ensemble*

Requires detailed map data.





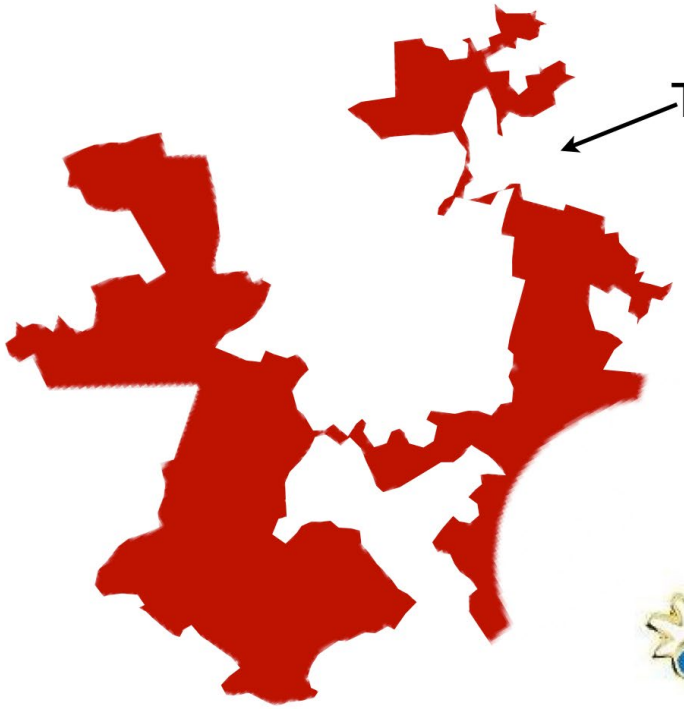
Images from Quantifying Gerrymandering Group

What if we don't have detailed map data? Or if we want a quick calculation?

- Can use a metric:
  - A number, calculated from a single map
- Kinds of metrics:
  - ~~Shape metrics~~
  - Partisan symmetry metrics
    - Partisan Bias
    - Mean-Median Difference
  - Election Outcome Metrics
    - Efficiency Gap
    - Declination
  - Geography and Election Outcome (GEO) Metric

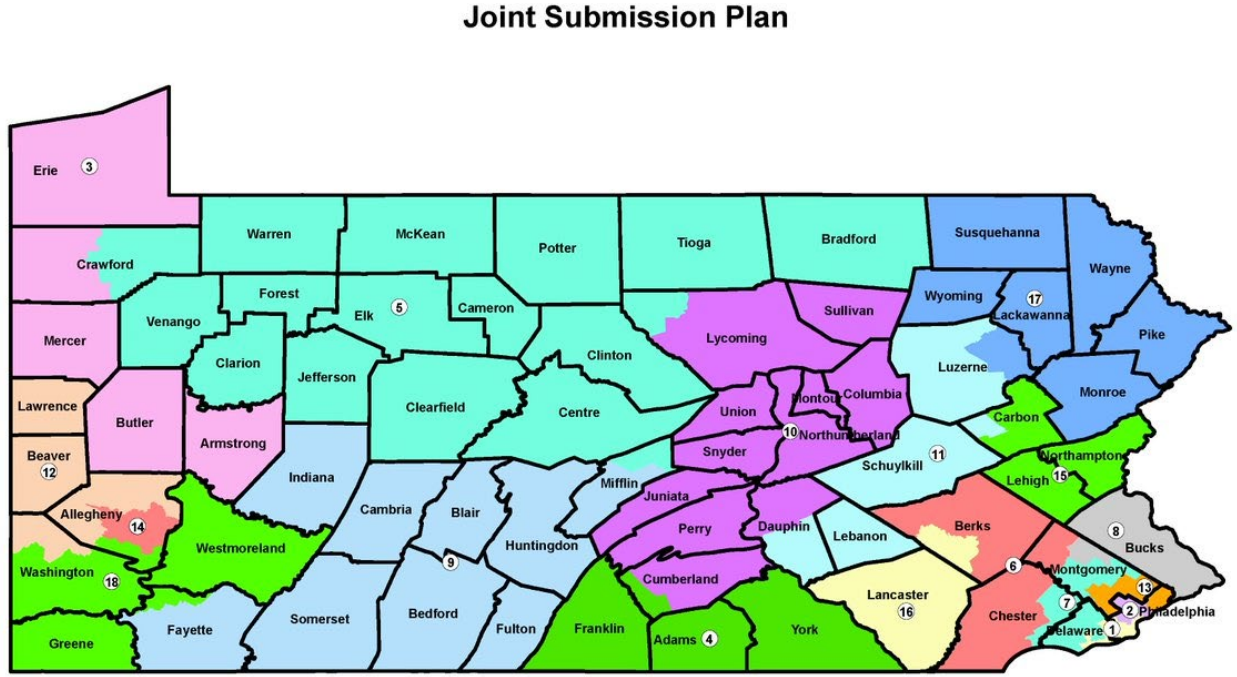
Shape Metrics: How irregular are district shapes?

Not effective at detecting gerrymanders

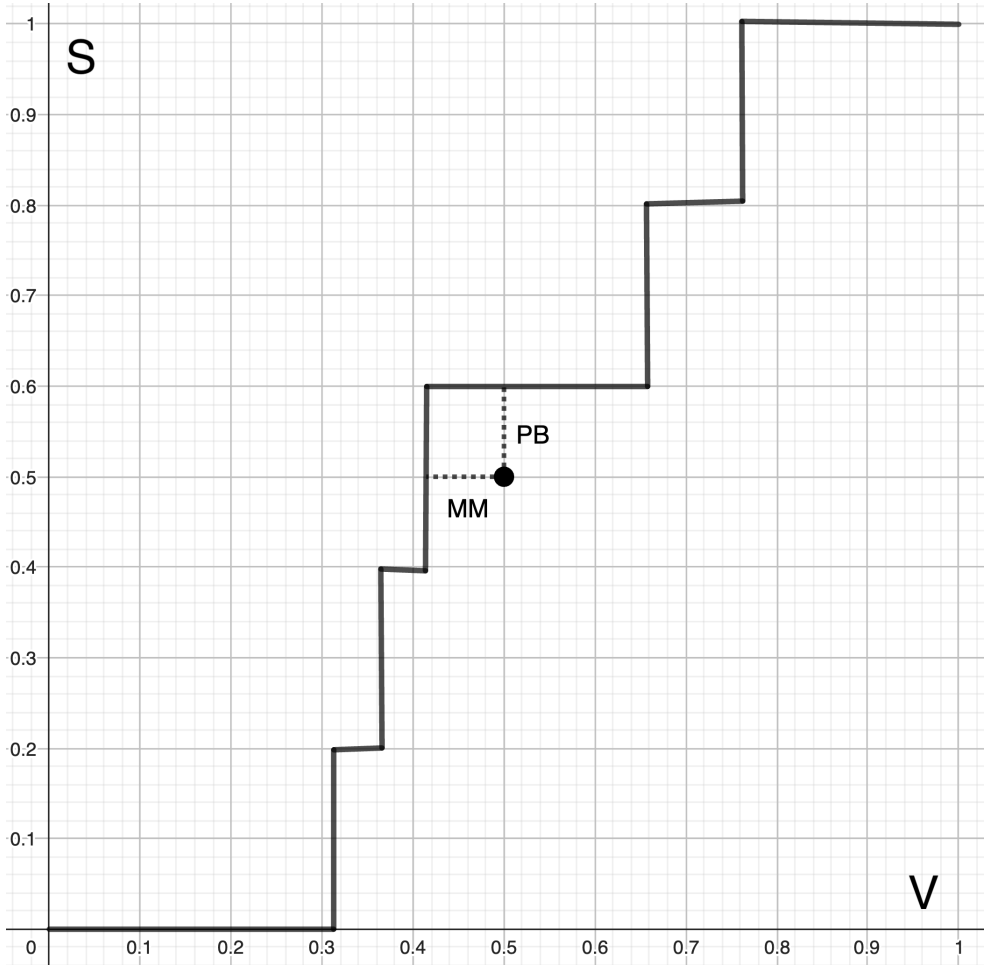


This Image depicts:

a. Goofy Kicking Donald Duck
b. A Laughing Girl kicking the Energizer Bunny
c. Korea and Japan
d. Pennsylvania's 7th Congressional District



Partisan Symmetry Metrics



Assume Uniform Partisan Swing:

- Start with single election outcome:
  - District vote shares

  $V_1, V_2, \ldots, V_n \qquad V = \frac{1}{n}\sum_{i=1}^{n} V_i$

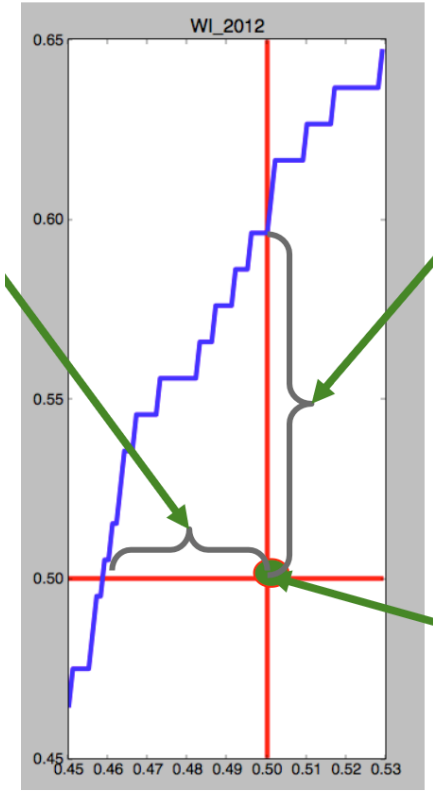
  vote share V, seat share S
- Assume swing for party A is uniform:

  $$V_i + s$$
- Get new V, S which is plotted on seats-votes curve

Partisan Symmetry Metric: Partisan Bias

District vote shares: $V_1, V_2, \ldots, V_n, \qquad \frac{1}{n}\sum_{i=1}^{m} V_i = V$

$$PB = \frac{1}{2}\left(\frac{\#V_i \text{ larger than } V}{n} - \frac{\#V_i \text{ smaller than } V}{n}\right)$$



How much of an advantage in seats the party would have if the vote were evenly split

(0.5, 0.5): the one point required be on any symmetric curve

Image: Metric Geometry Gerrymandering Group

Partisan Symmetry Metric: Mean-Median Difference

District vote shares: $V_1, V_2, \ldots, V_n$

$$MM = \text{median}\{V_1, V_2, \ldots, V_n\} - \text{mean}\{V_1, V_2, \ldots, V_n\}$$

How far the party can fall from a majority of votes and still get a majority of seats

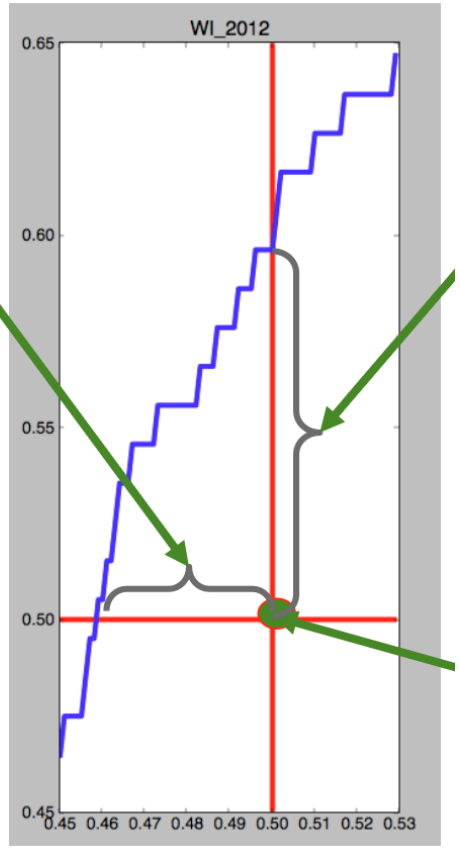


Image: Metric Geometry Gerrymandering Group

Election Outcome Metrics:

Efficiency Gap (Stephanopolous and McGhee, 2018), Declination (Warrington, 2019)

$$EG = \frac{\text{wasted votes for party } A - \text{ wasted votes for party } B}{\text{total votes}}$$

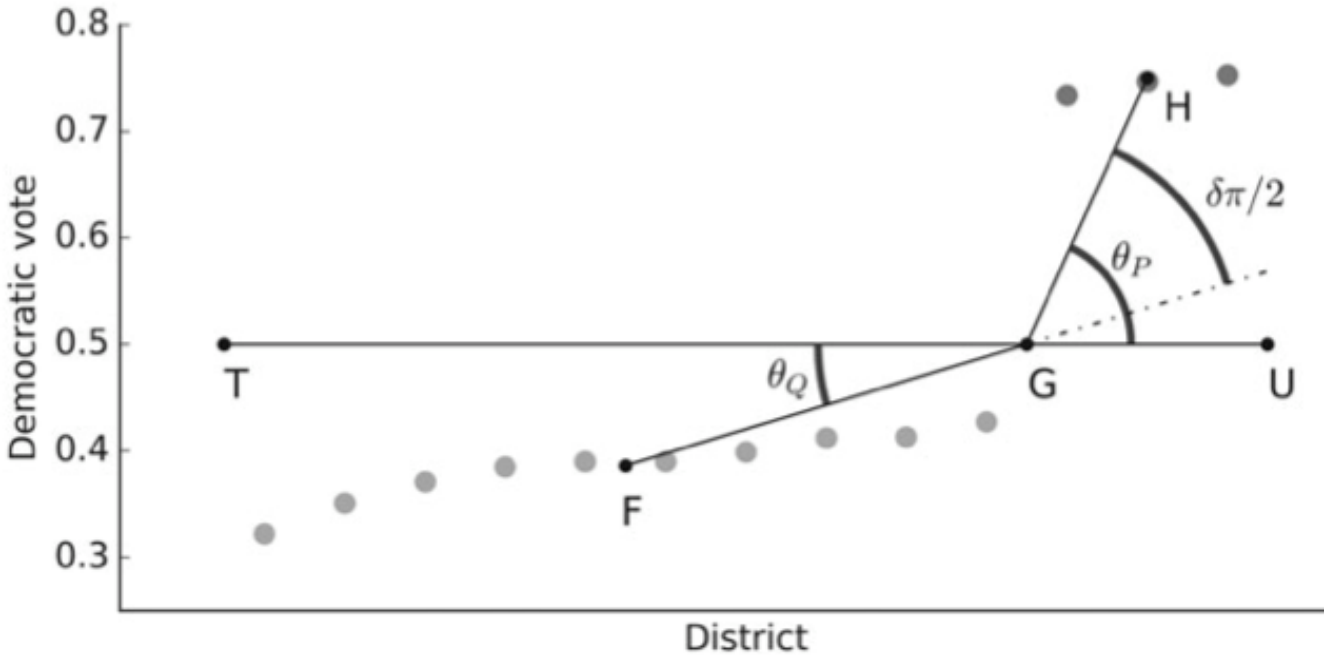


Image: G. Warrington

Geography and Election Outcome (GEO) metric

(Campisi, Ratliff, Somersille, *V*, 2022)

For each party: *Count* additional districts that *could be made competitive* for that party

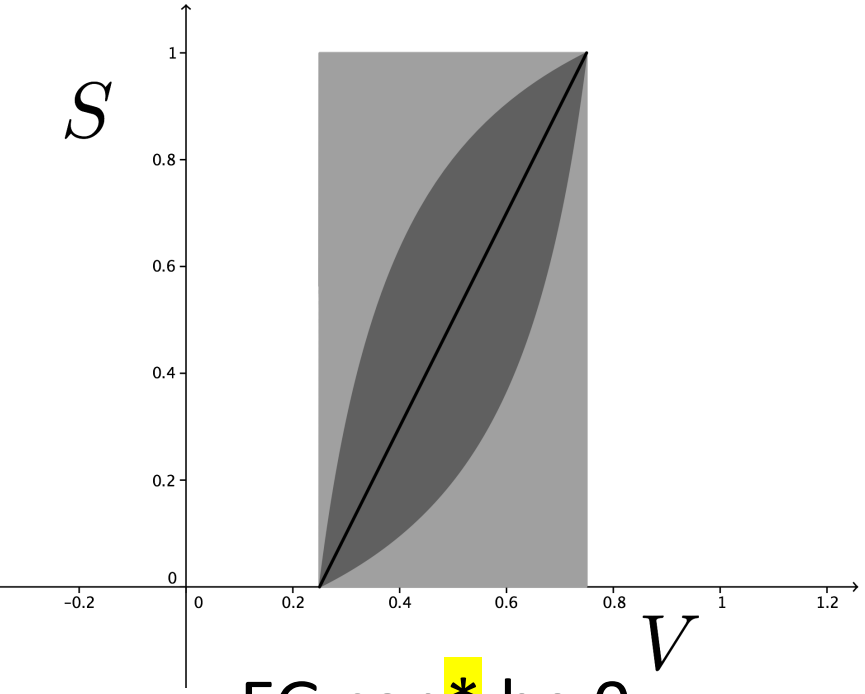
District 1 10%	District 2 10%
District 3 46%	District 4 46%
District 5 61%	District 6 61%
District 7 46%	District 8 46%
District 9 62%	District 10 62%

District 1 62%	District 2 62%
District 3 61%	District 4 61%
District 5 10%	District 6 10%
District 7 46%	District 8 46%
District 9 46%	District 10 46%

GEO metric can distinguish between two maps with the same district vote shares, but different adjacencies

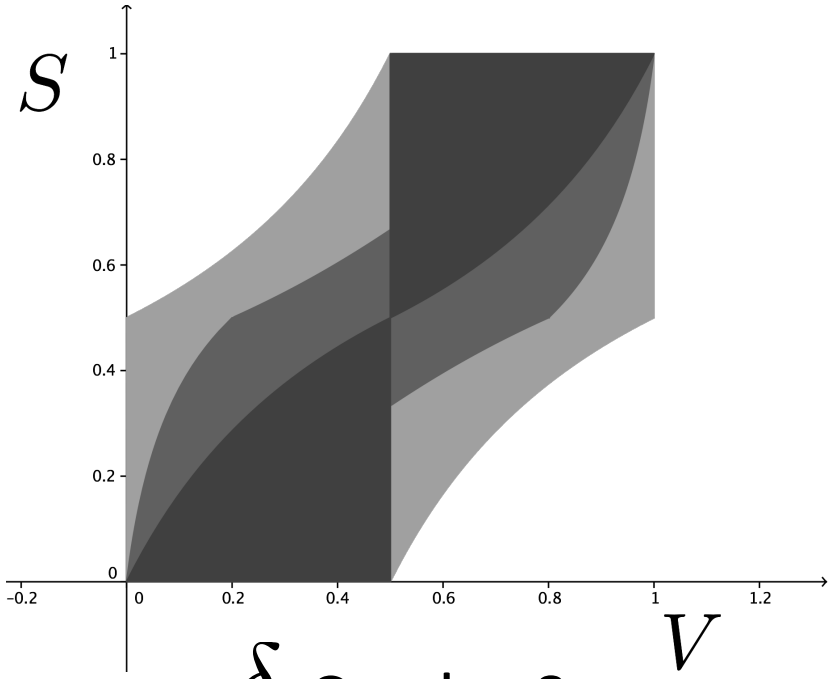
Partisan Symmetry and Election Outcome Metrics

Darkest grey: metric can be 0 for (V, S) when turnout is equal in all districts
Middle grey: metric can be 0 for (V, S) when max/min turnout no more than 4
Lightest grey: metric can be 0 for (V, S), no restriction on turnout



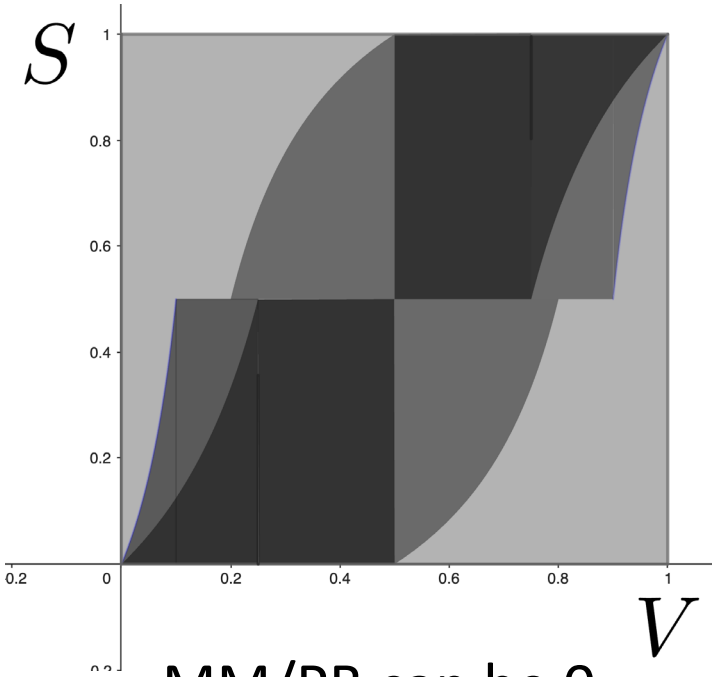
EG can* be 0

(V. 2018)



δ Can be 0

(Campisi, Padilla, Ratliff, V. 2019)

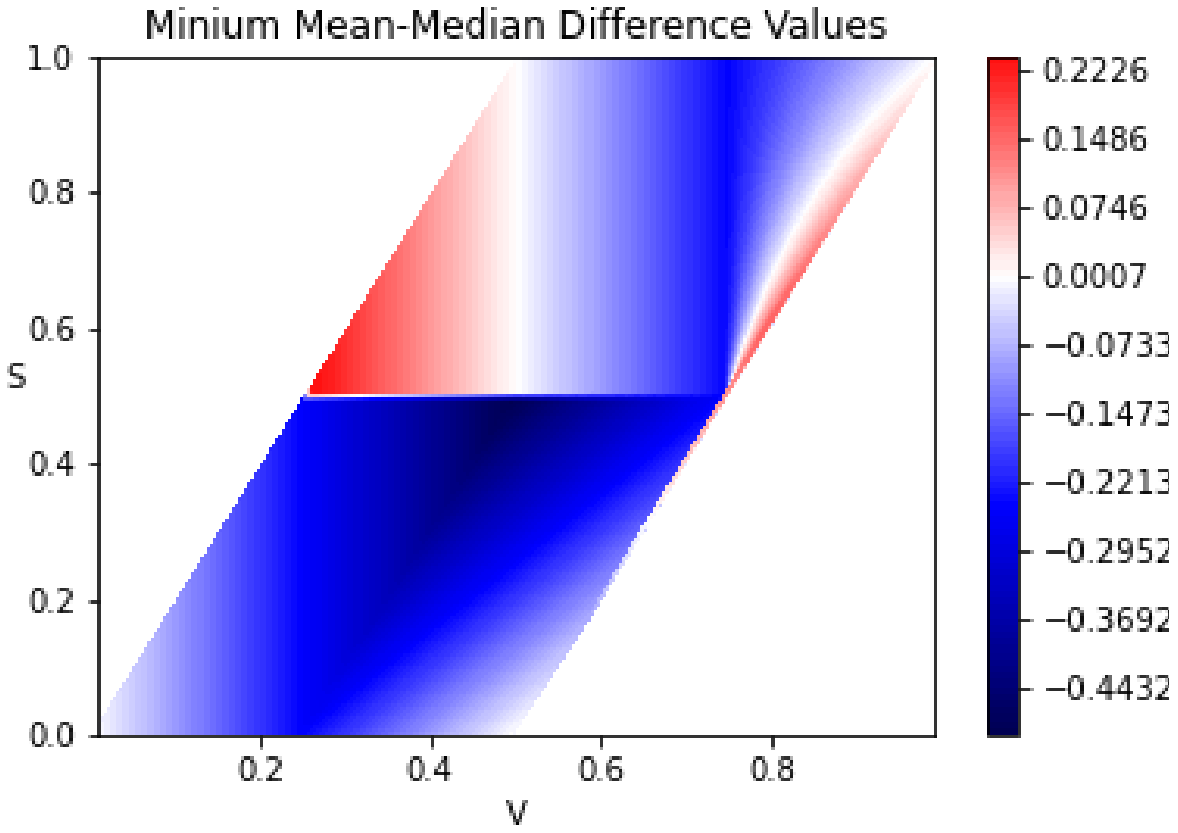


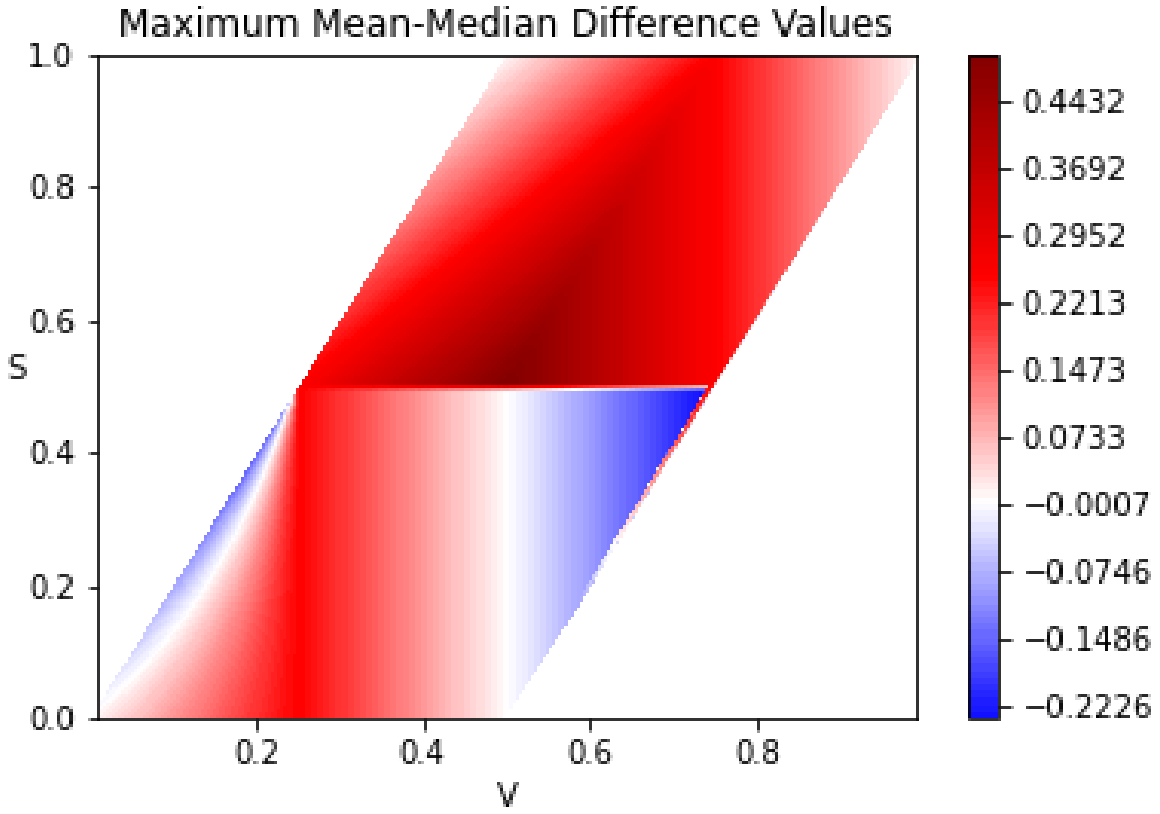
MM/PB can be 0

(V. 2024)

Min and Max Values for Mean-Median Difference

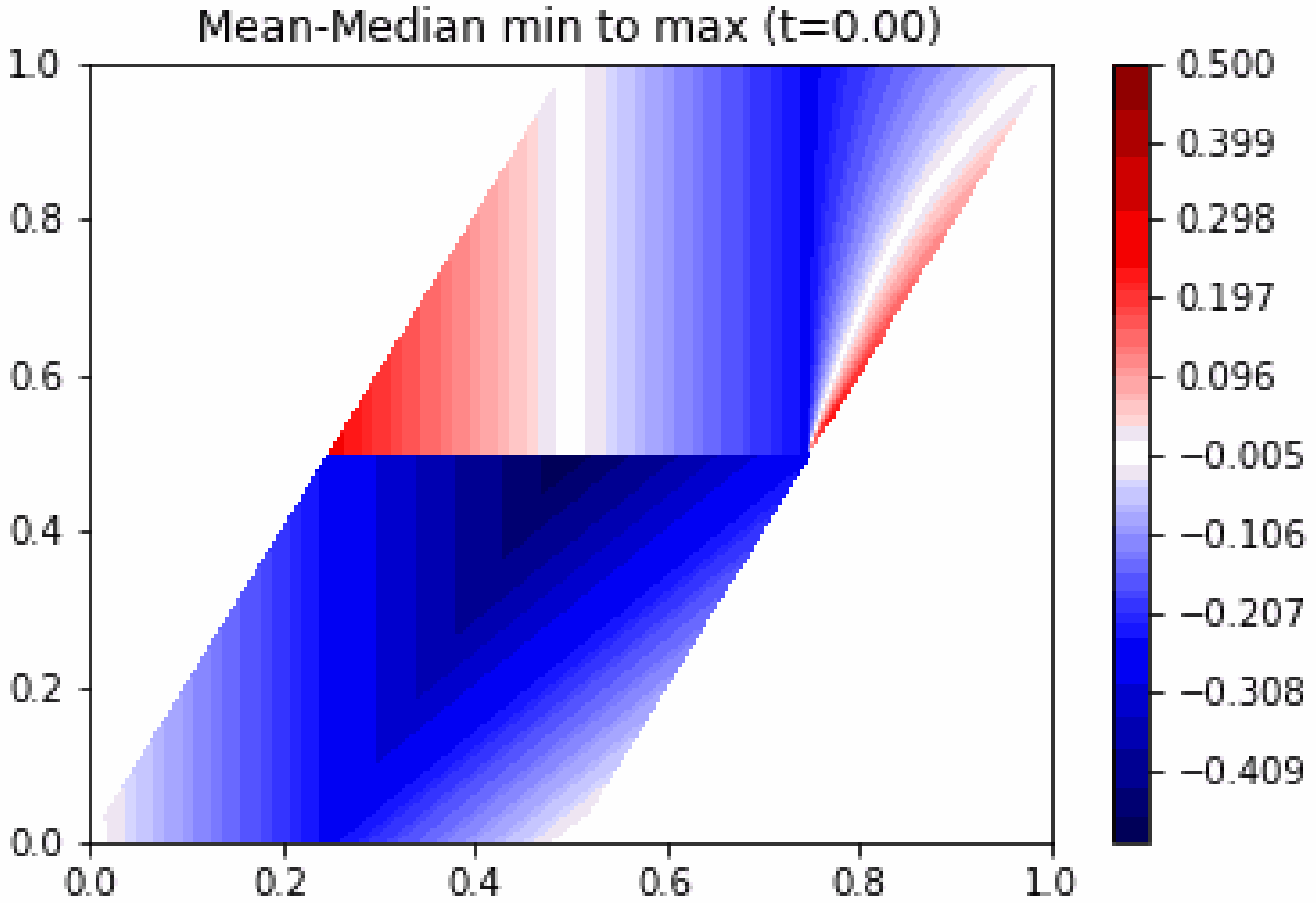
(partisan symmetry metric)





Min to Max Value for Mean-Median Difference

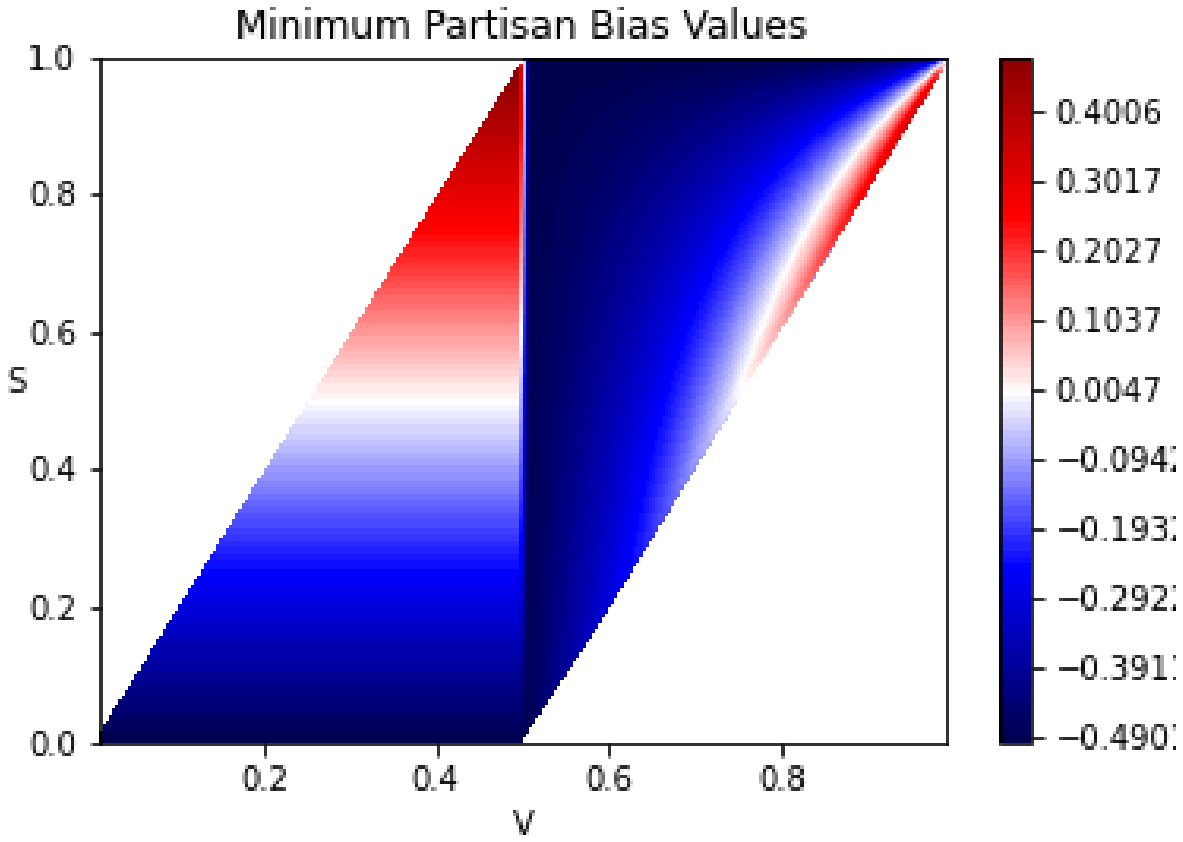
(partisan symmetry metric)

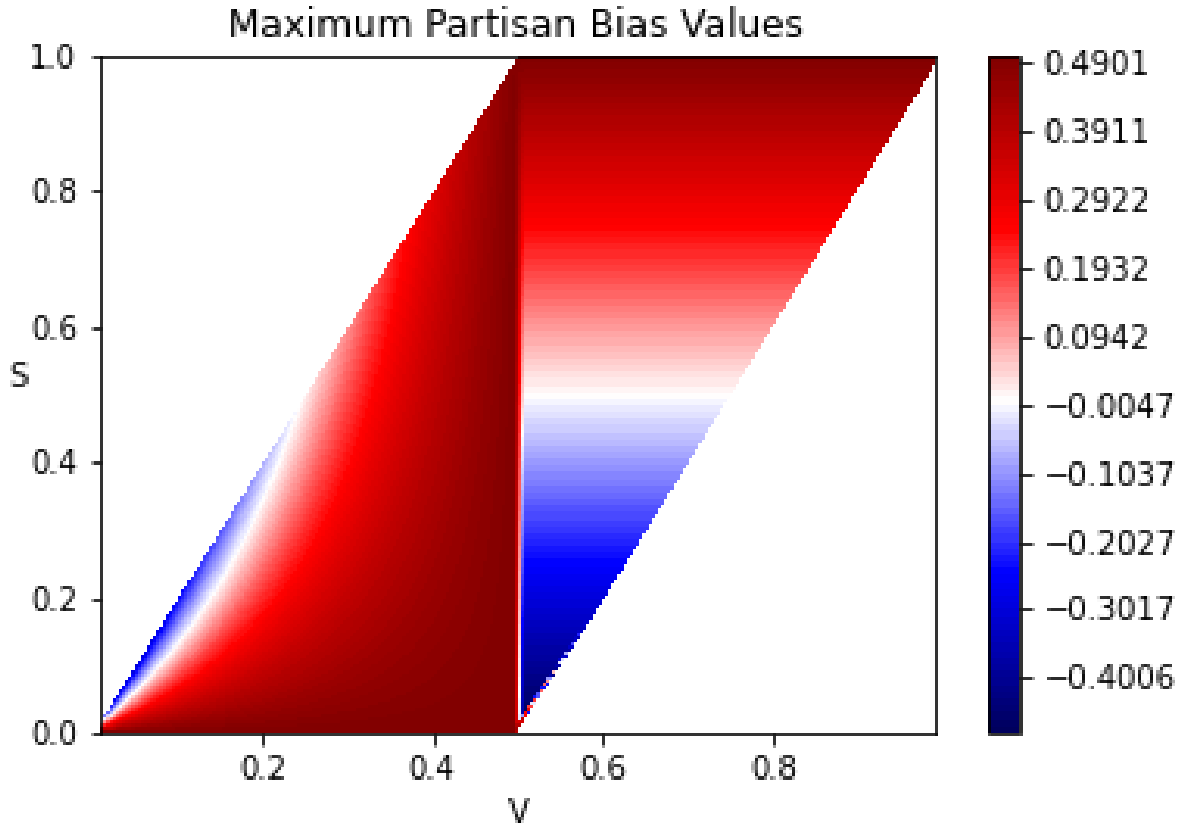


(V. 2024)

Min and Max Values for Partisan Bias

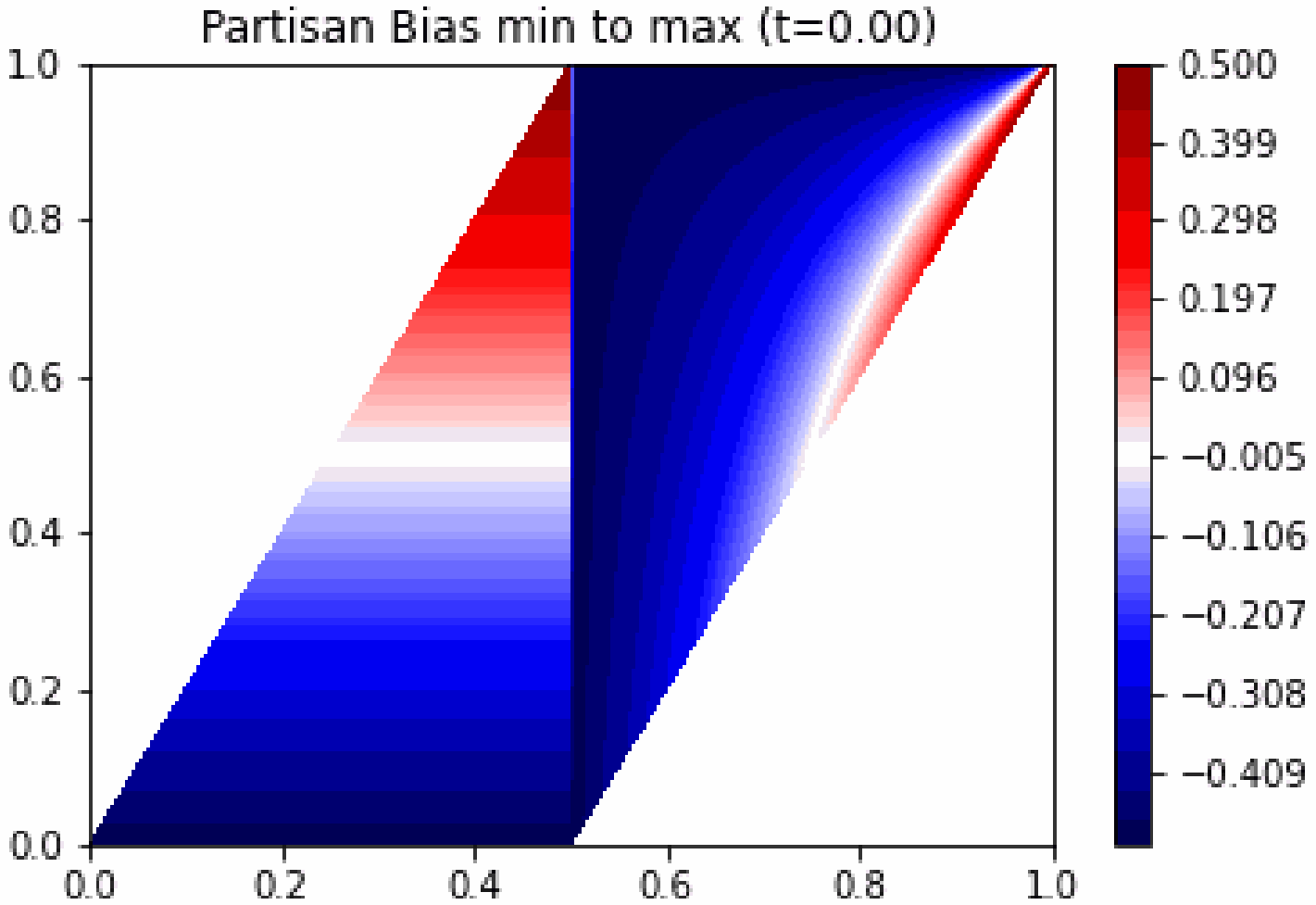
(partisan symmetry metric)





Min to Max Value for Partisan Bias

(partisan symmetry metric)



(V. 2024)

Which metrics can be “gamed”?

- We know how metrics *can* act in theory
- How do they act on real data? Can they be gamed?
  - Can we find an extreme map to which the metric gives a “passing score”?
- Definition of “extreme map”
  - A map with an extreme number of districts won by party A
  - Ultimately partisan gerrymandering is done with a focus on seats won
  - We use number of seats won as the standard by which we judge the gerrymandering metrics

Short Bursts

- Developed by S. Cannon et al to find “majority minority” districts
- Take a small number of steps (s=10) in a random walk on metagraph of redistricting maps
- Within that “short burst,” find the most recent map with the largest number of majority-minority districts
- That map is the seed of the next “short burst” of size 10.
- Repeat

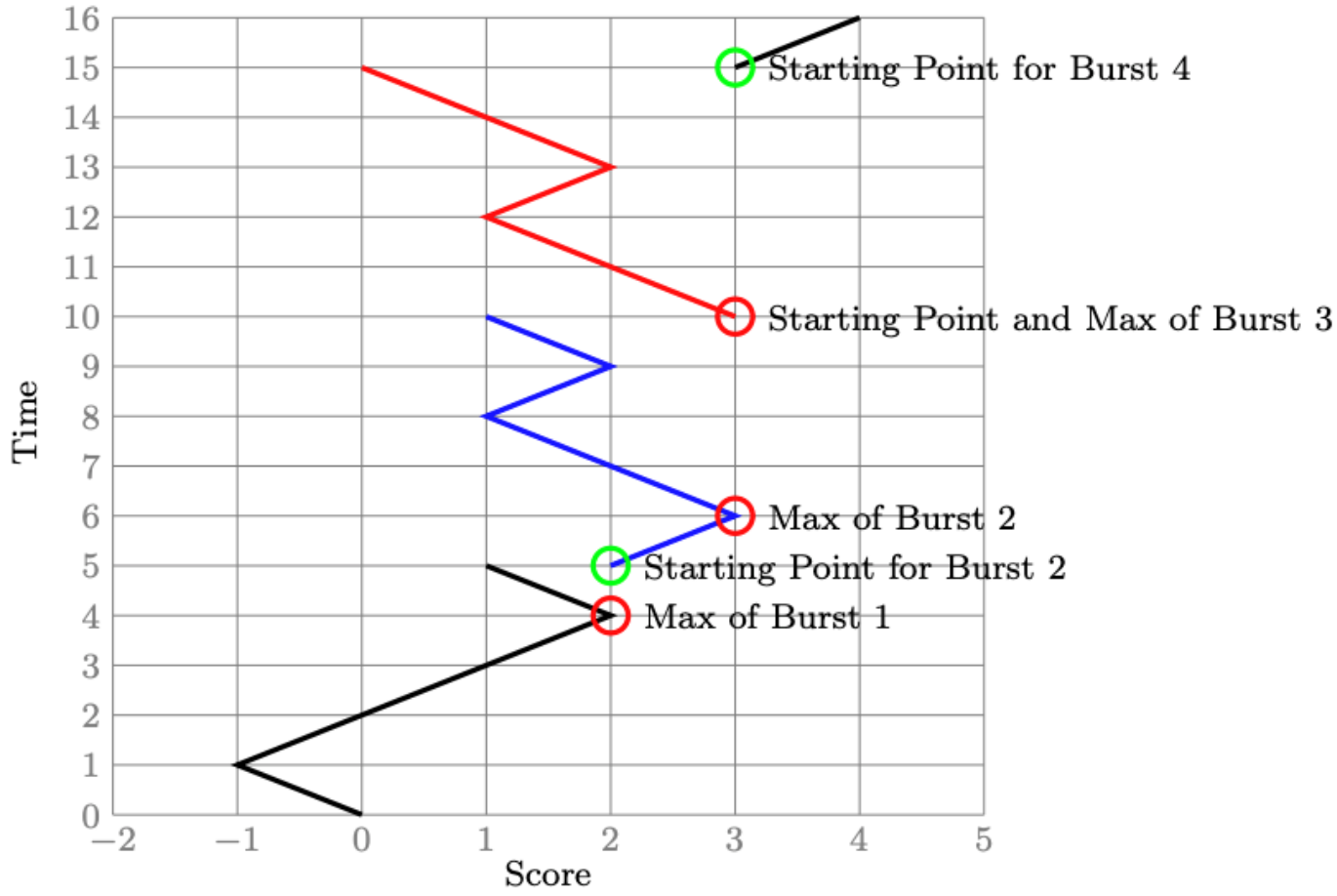


Image: S. Cannon et al

Our Usage of Short Bursts

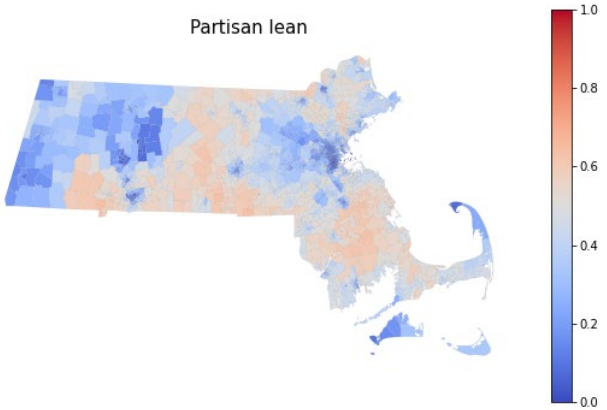
(Ratliff, Somersille, V. 2024)

- Trial 1: fix bounds
  - Fix a metric to be within "reasonable bounds" (based on bounds proposed by creators of Efficiency Gap)
  - Run a short burst to maximize districts won by Democrats (or Republicans)
  - What number of districts won is considered acceptable by that metric?

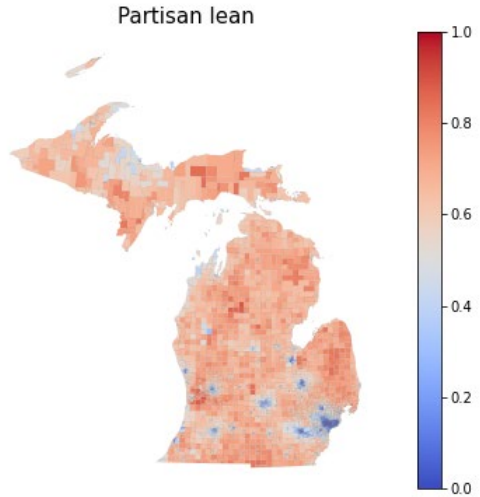
- Trial 2: no bounds
  - No constraints on any metric
  - Run a short burst to maximize districts won by Democrats (or Republicans)
  - How large can number of districts won go?
  - What do the metric values do?

Note: We adapted S. Cannon's team's code, which uses MGGG's GerryChain

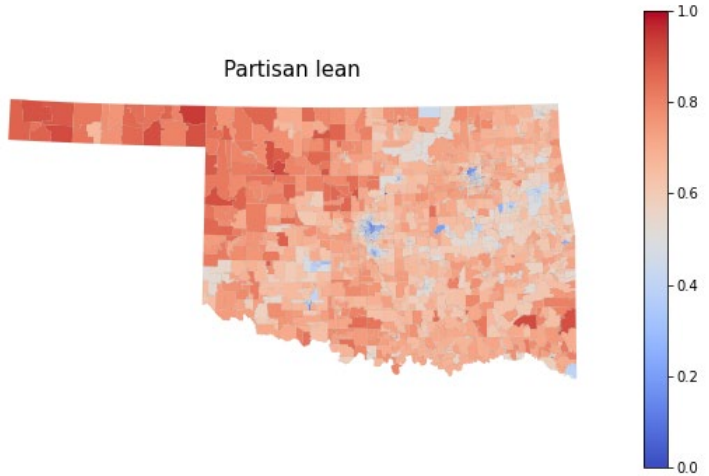
States Analyzed



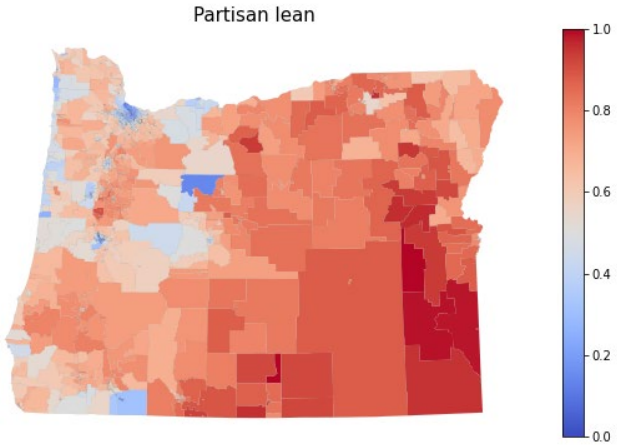
OK (blue, homogeneous)



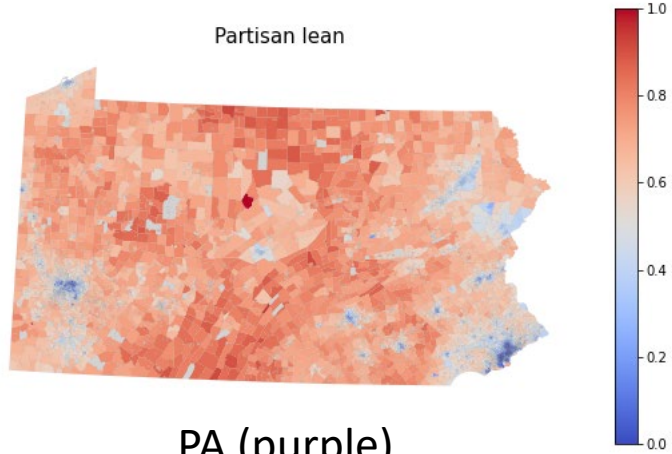
MI (purple)



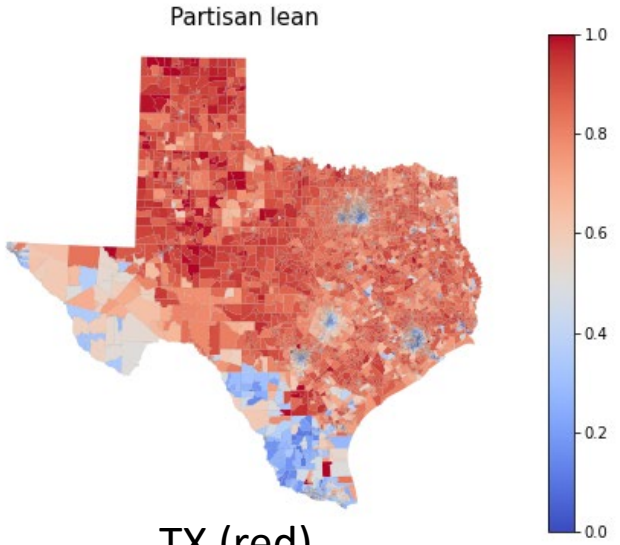
OK (red, homogeneous)



OR (blue)



PA (purple)



TX (red)

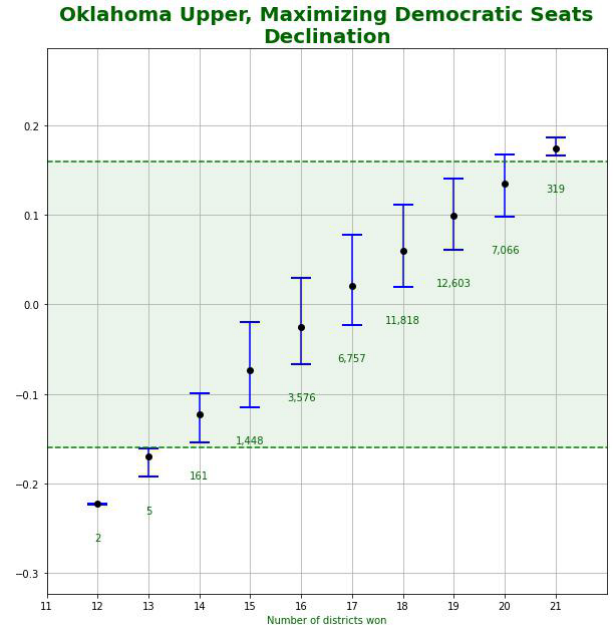
Results of Trial 1: fix bounds (Gameability)

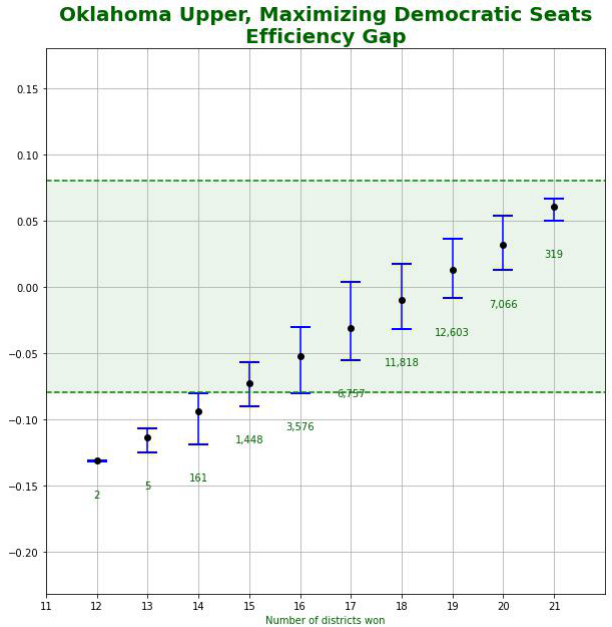
	Num	Dem Max					Rep Max				
	Seats	None	Dec	EG	GEO	MM	None	Dec	EG	GEO	MM
MA cong	9	9			9	9	1			1	1
MA lower	160	**148**	142	136	134	148	**41**	38	41	42	41
MA upper	40	40	40	35	36	40	10	8	10	10	10
MI cong	13	10	8	8	8	10	12	8	8	10	12
MI lower	110	**59**	60	60	60	59	**78**	62	65	69	78
MI upper	38	24	22	22	21	24	29	22	22	25	29
OK cong	5	2	1		1	2	5	4		5	5
OK lower	101	41	40	40	39	41	83	73	71	76	83
OK upper	48	21	20	21	20	21	43	37	35	39	43
OR cong	5	4	3	3	4	4	4	2	2	3	4
OR lower	60	40	37	38	36	40	39	32	33	36	39
OR upper	30	22	19	19	19	22	22	16	16	18	22
PA cong	18	11	10	9	9	10	15	11	11	12	15
PA lower	203	**98**	98	98	98	98	**145**	120	123	130	145
PA upper	50	27	27	27	27	27	41	30	31	33	41
TX cong	36	16	13	13	13	16	34	31	30	31	34
TX lower	150	58	58	57	59	58	124	109	123	113	124
TX upper	31	14	11	12	11	14	29	27	26	27	29

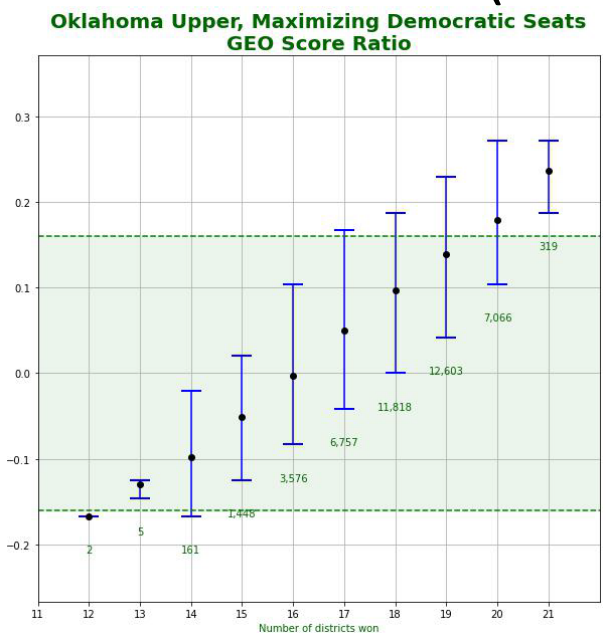
Conclusions for Trial 1: fix bounds (Gameability)

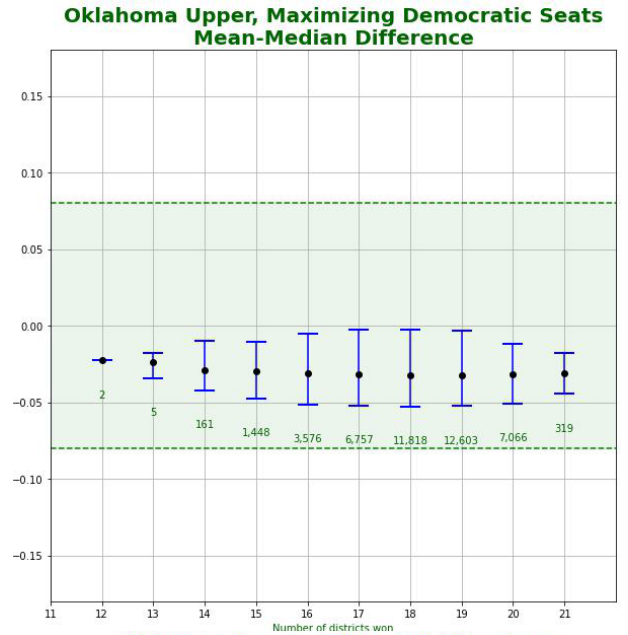
- Every metric can be gamed, but *especially MM*
  - PA congressional was the only map where restricting MM also restricted number of seats won (by Dems)
- On some maps, restricting some metrics pushed the local search towards maps with even *higher* numbers of districts won!
- In general, Republicans more restricted, except in Massachusetts
  - Perhaps due to political geography?

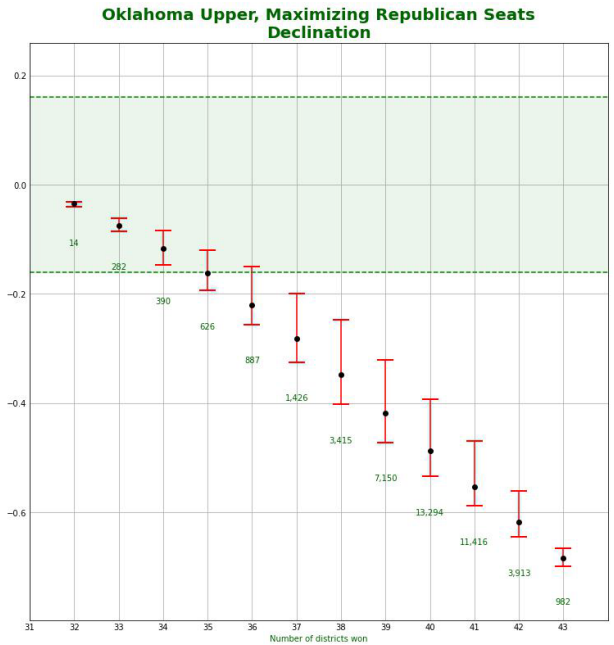
Results of Trial 2: no bounds (sample: OK)

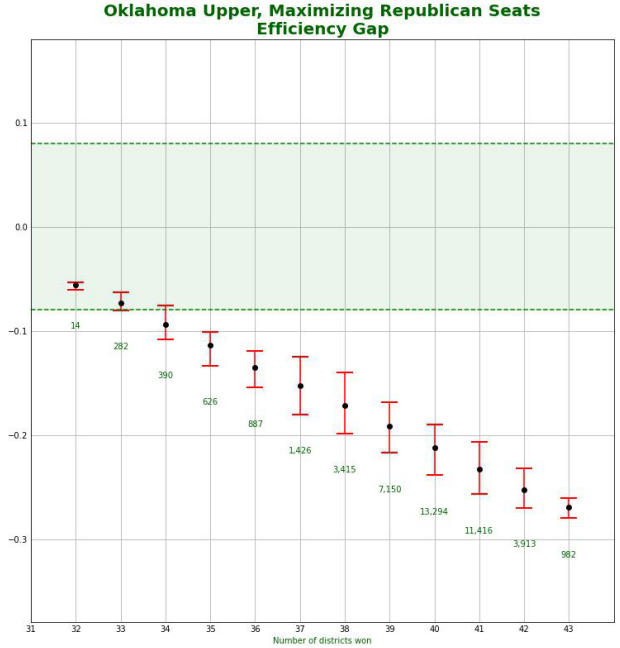


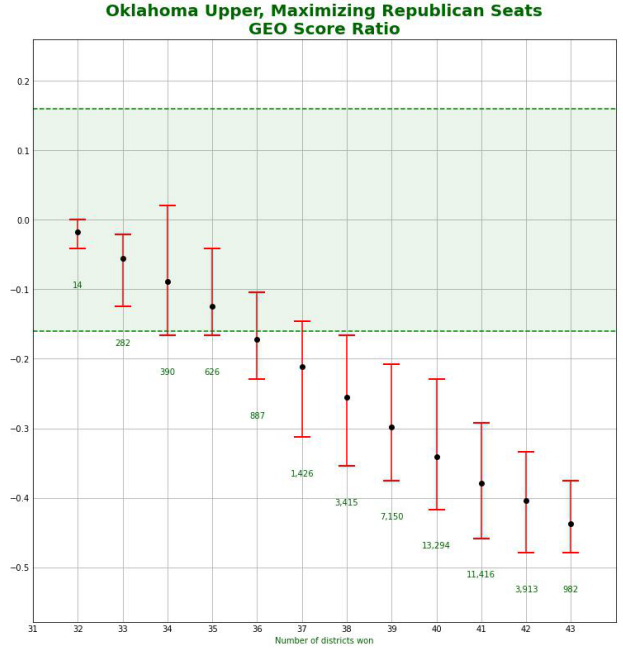


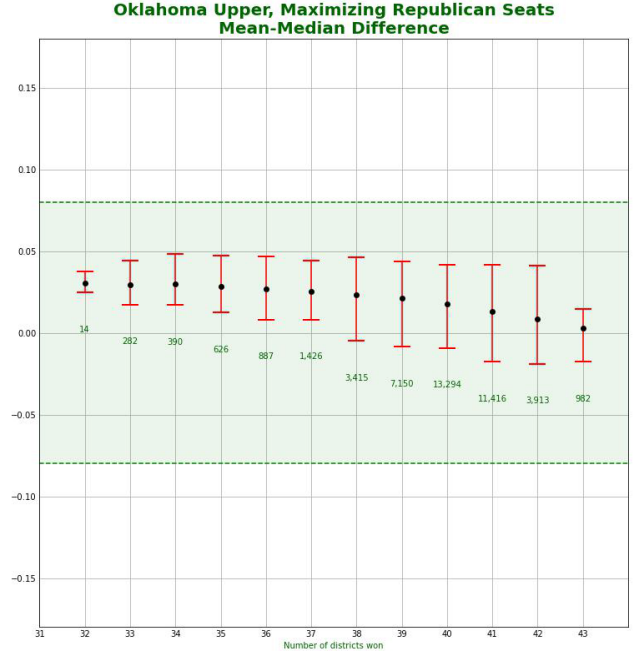






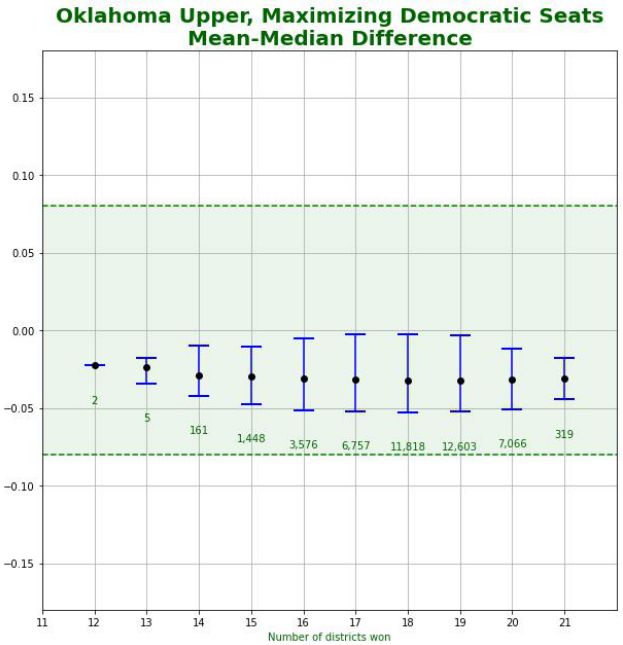


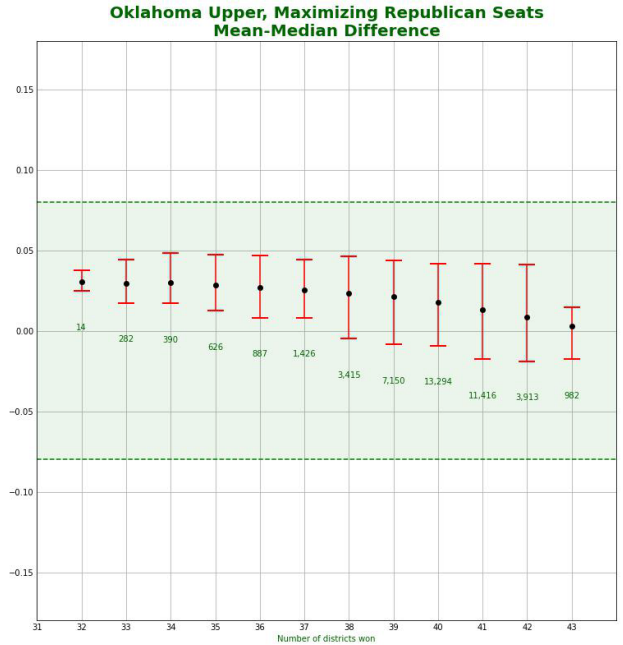


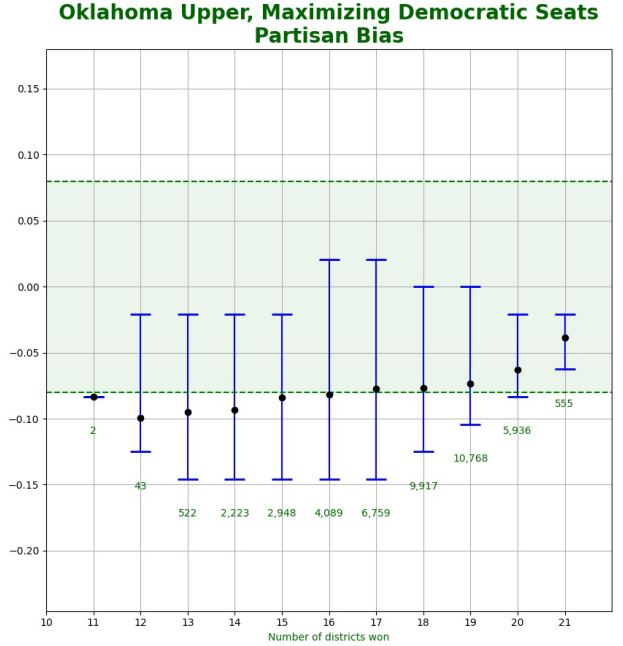


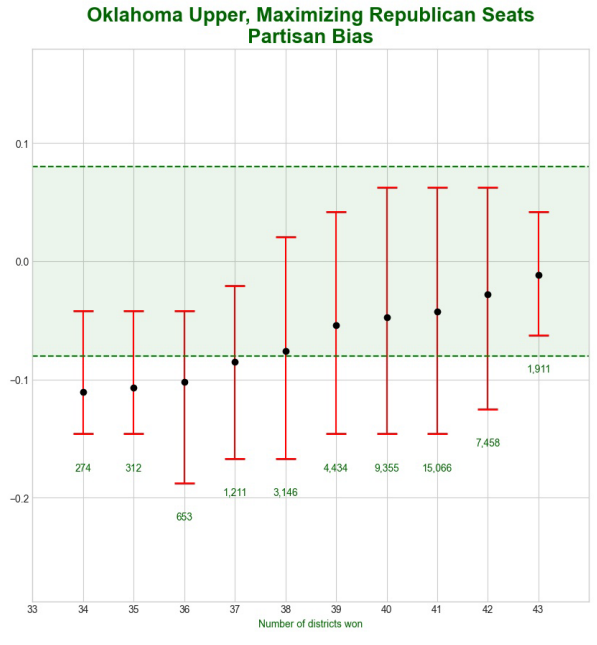
Conclusions for Trial 2: no bounds

- As expected, most metrics fell outside of the “acceptable range” at some point
  - Except MM
- Republican-won districts tend to fall outside of the “acceptable range” earlier
- *Values* for MM don’t change much, as S increases! (same effect with PB)







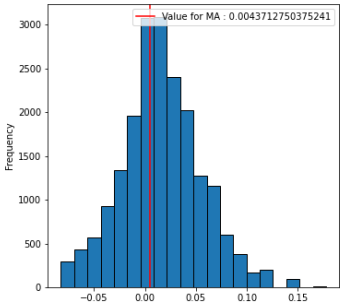


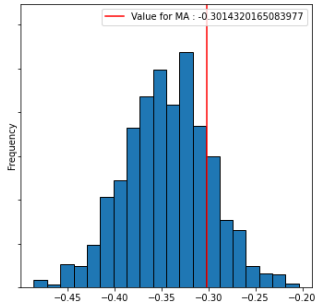
How does each metric compare to "districts won" on a neutral ensemble?

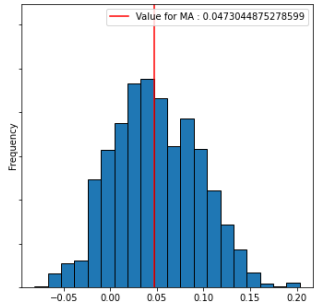
- We use number of seats won as the standard by which we judge the gerrymandering metrics
- Does any metric pick out a gerrymander that "districts won" doesn't?
  - False positive
- Does any metric miss a gerrymander that other metrics catch?
  - False negative

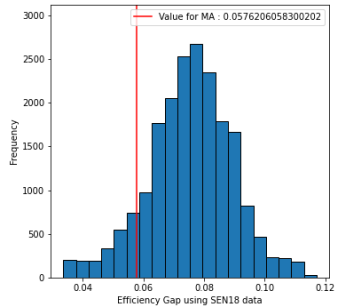
Note: We used MGGG's GerryChain to create neutral ensembles

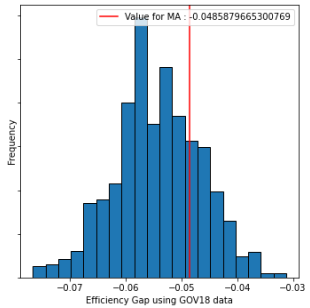
Metrics on Neutral Ensemble
(mostly agree with districts won)

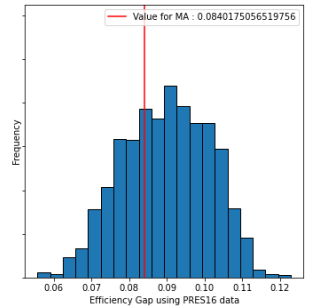


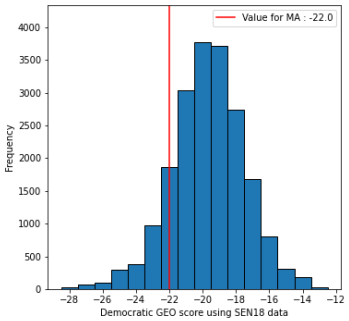


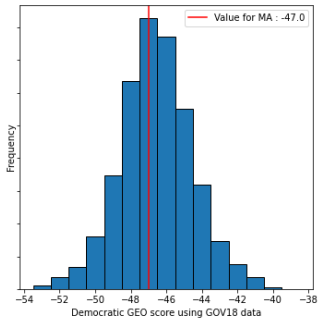


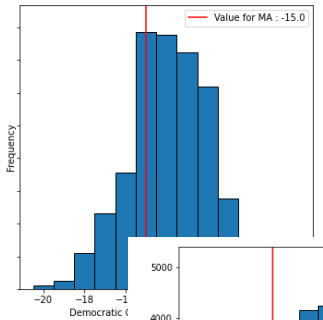


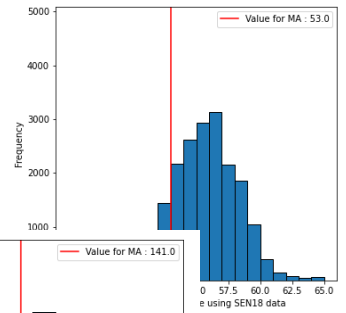


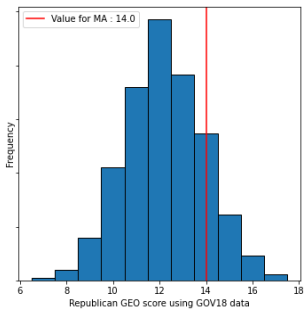


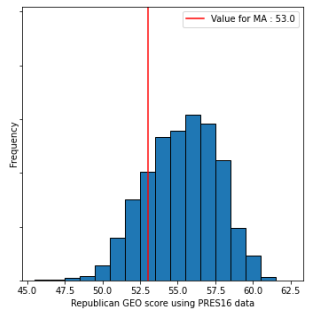


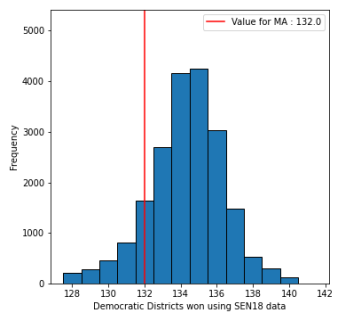


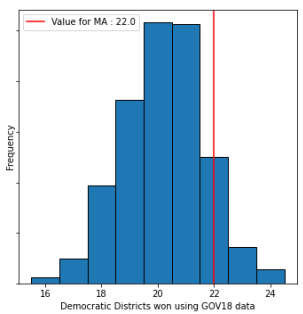


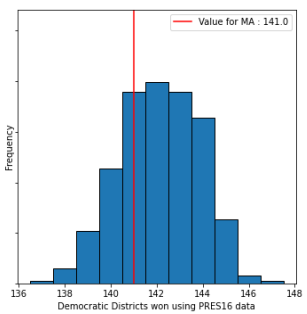


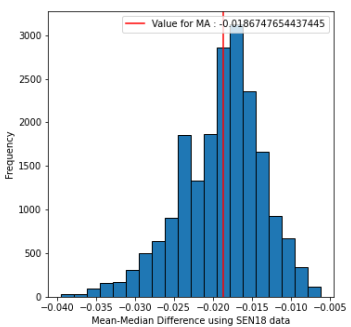


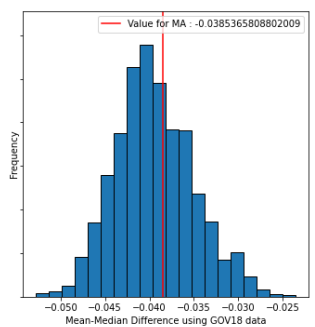


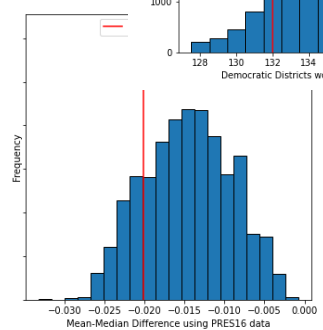


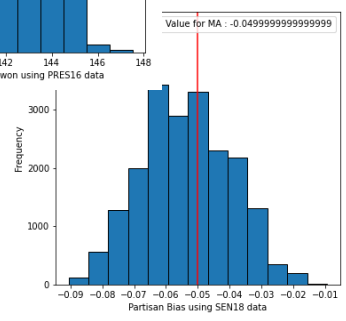


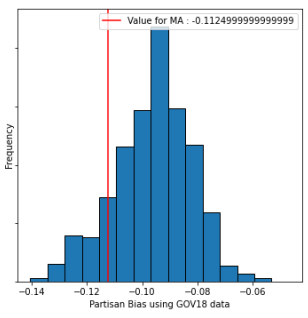


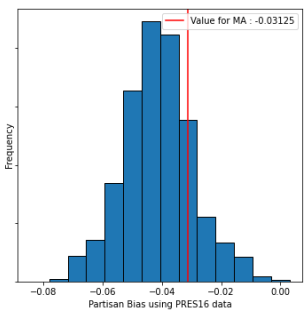






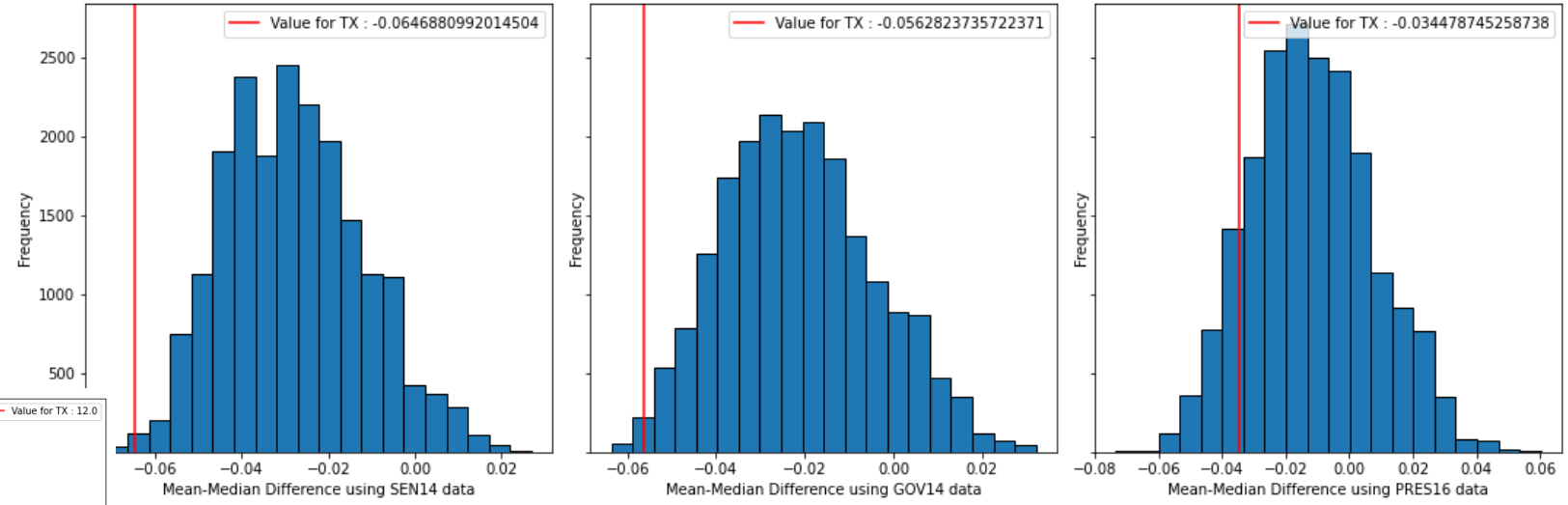


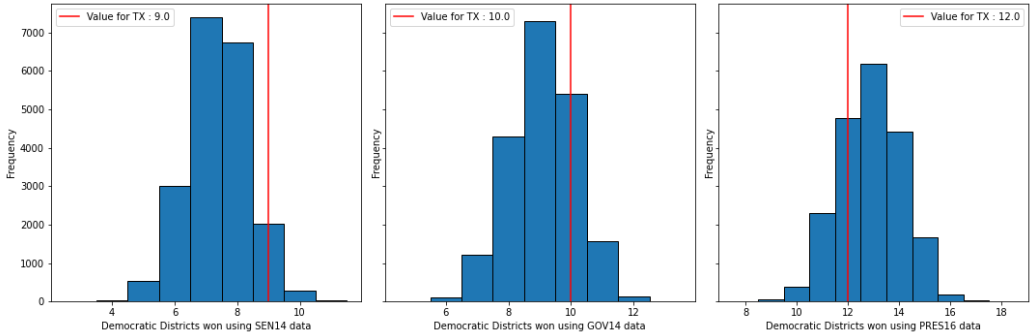


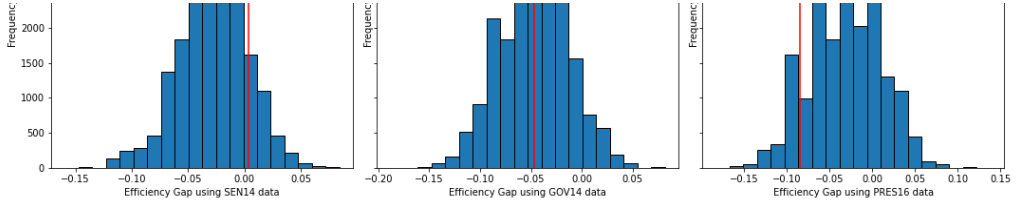


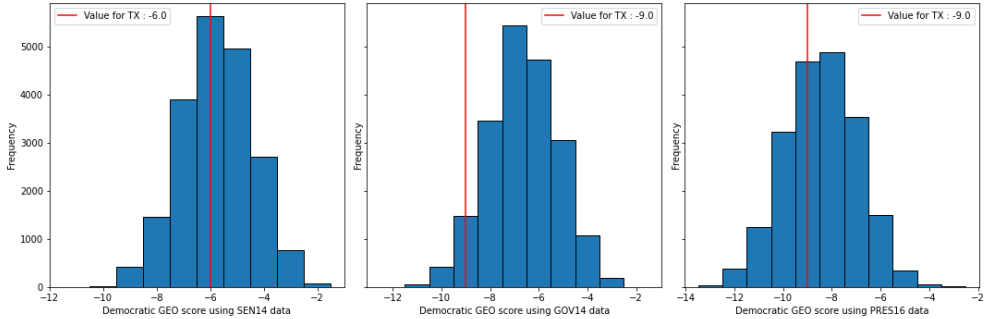
MM: false positive

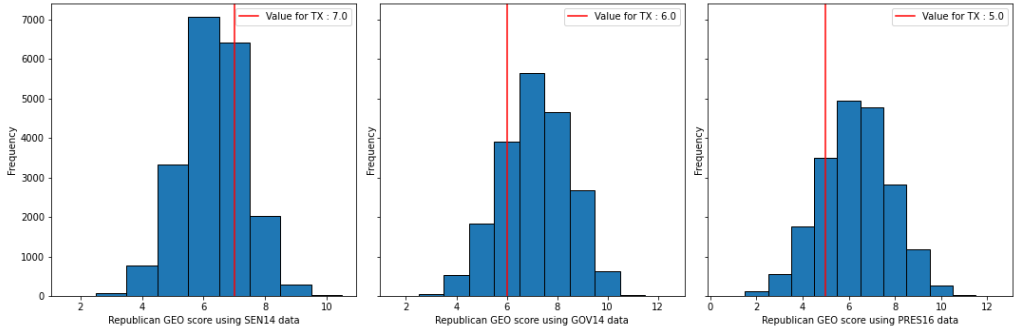
(on two election data for TX senate)

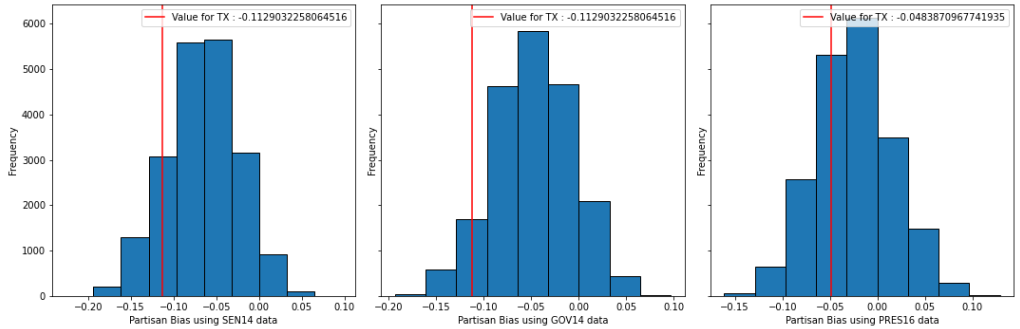


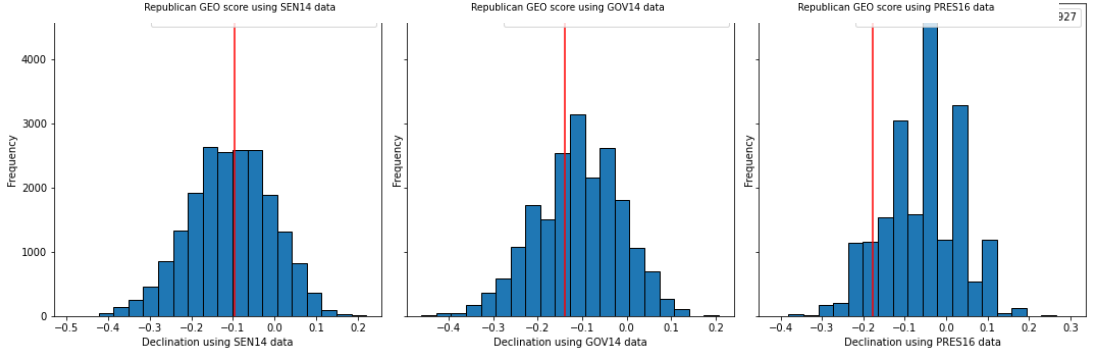




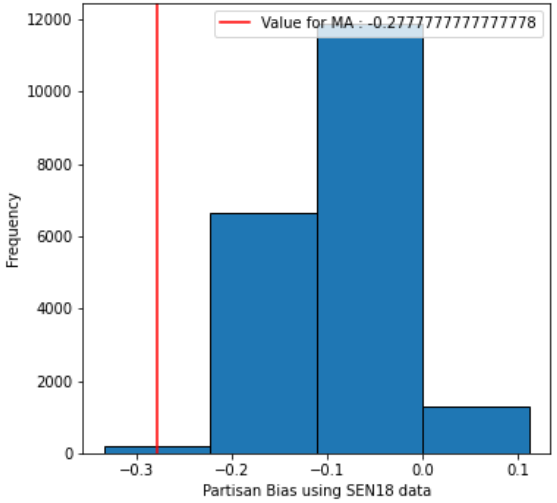


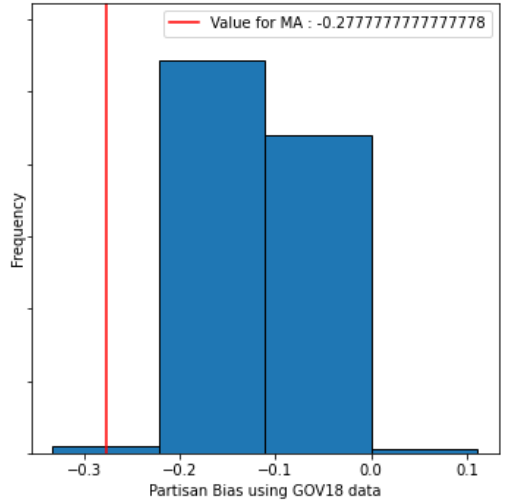


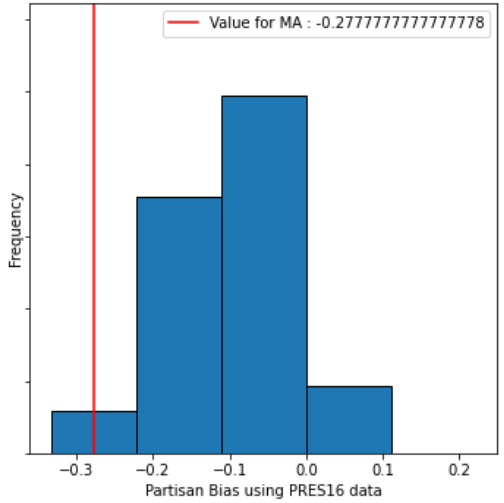


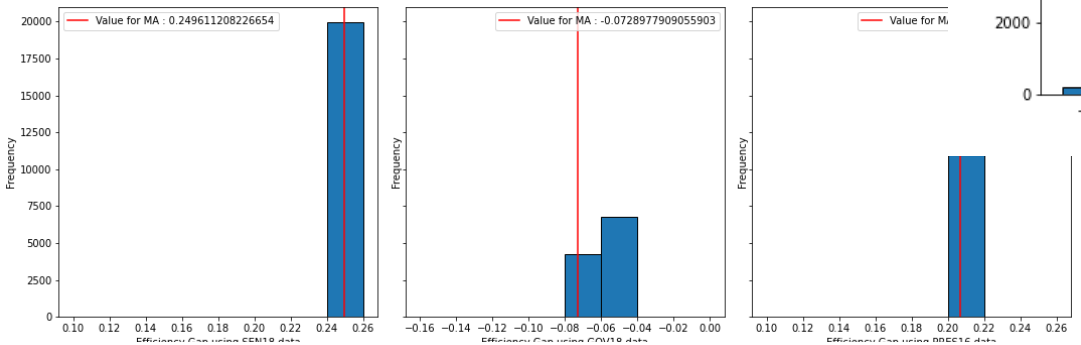


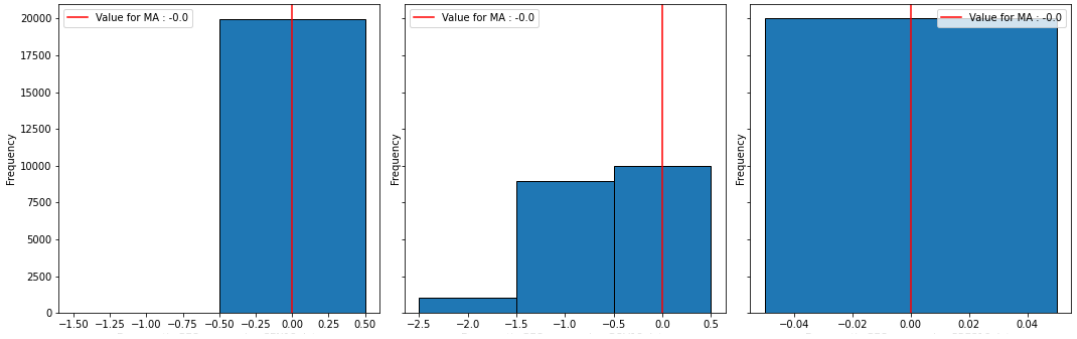
PB: false positives
(on MA congressional)

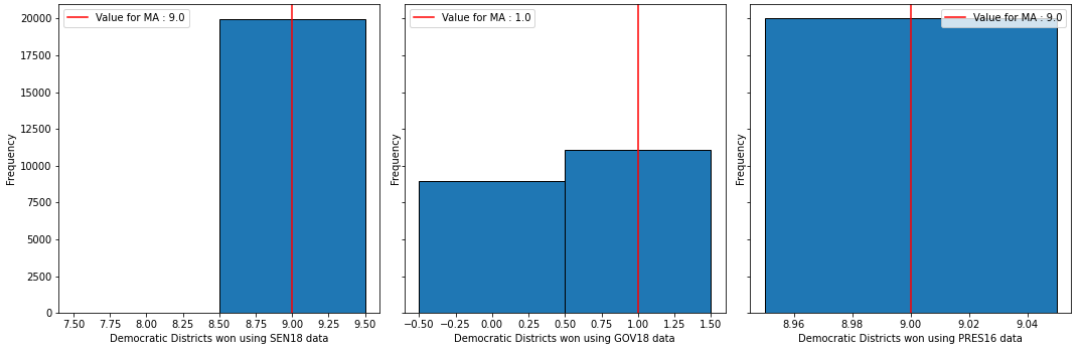


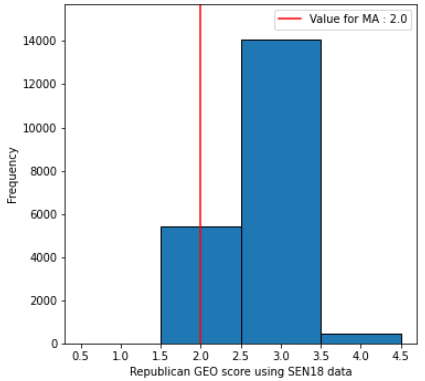


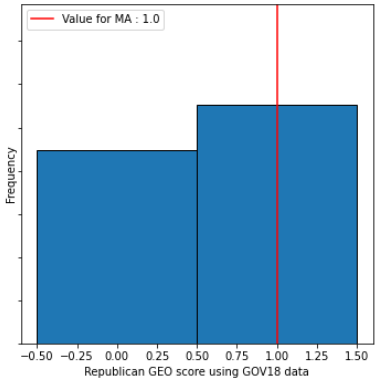


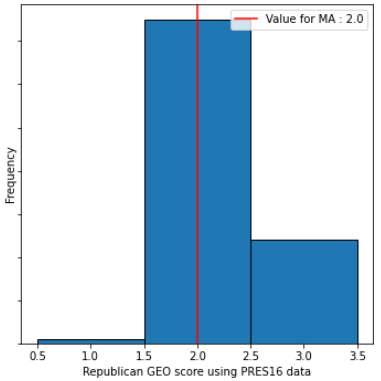


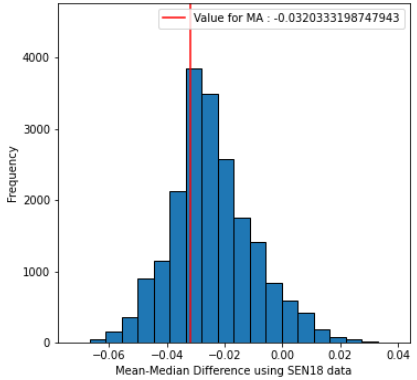


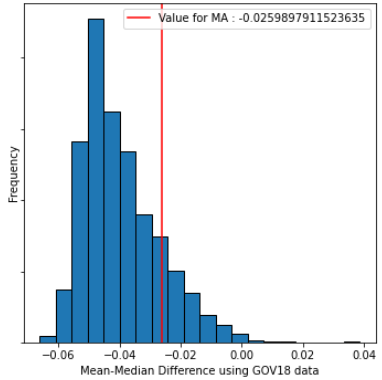


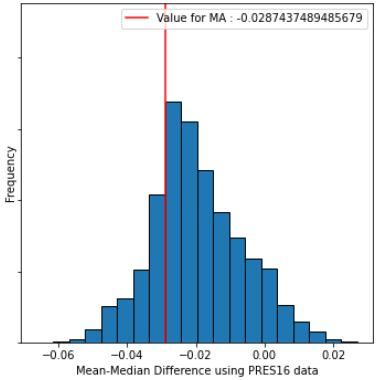




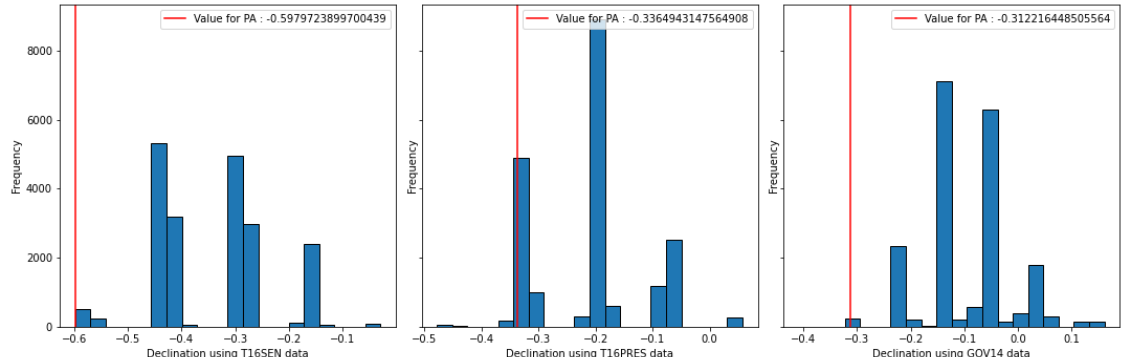


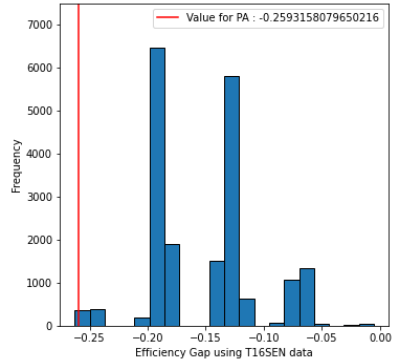


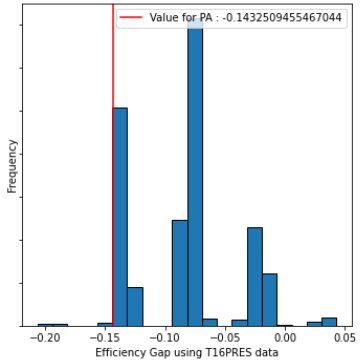


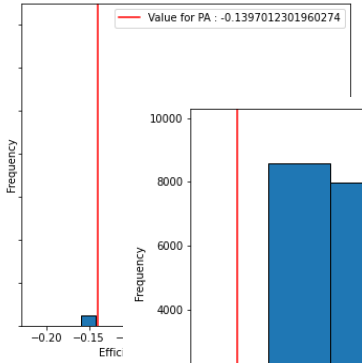


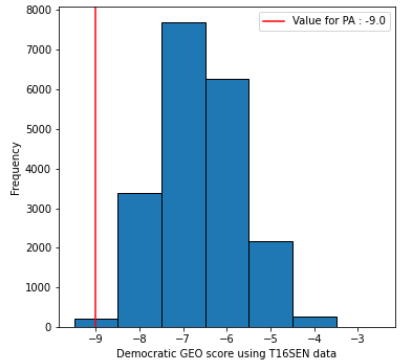
MM and PB: false negatives (on PA congressional)

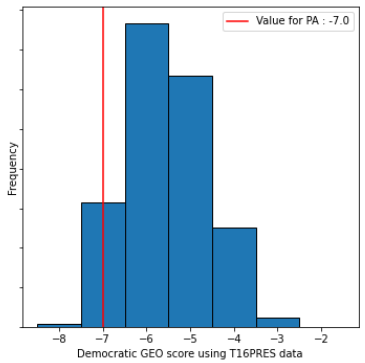


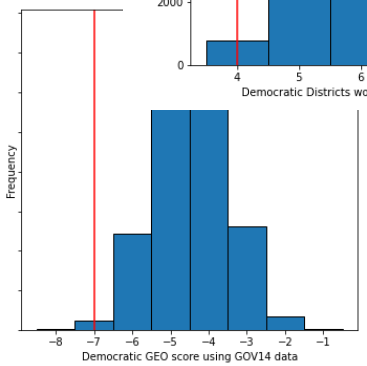


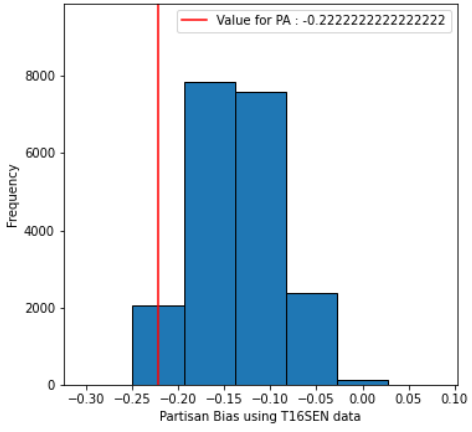


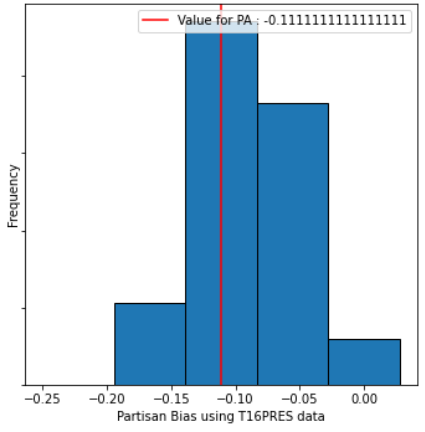


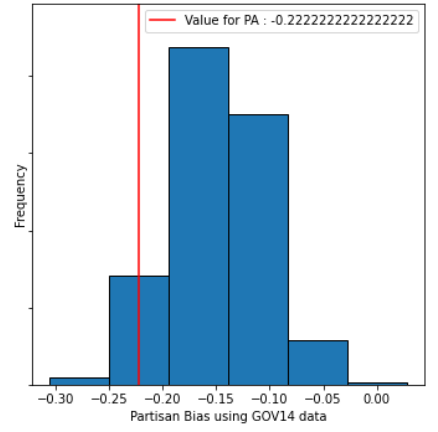


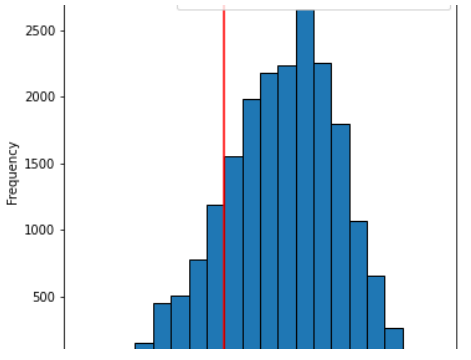


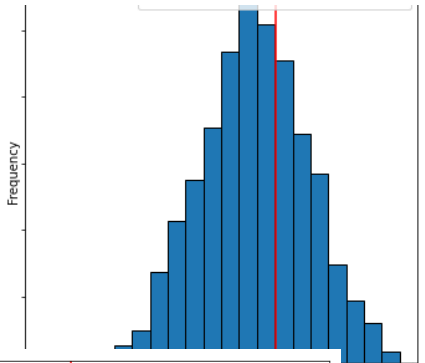


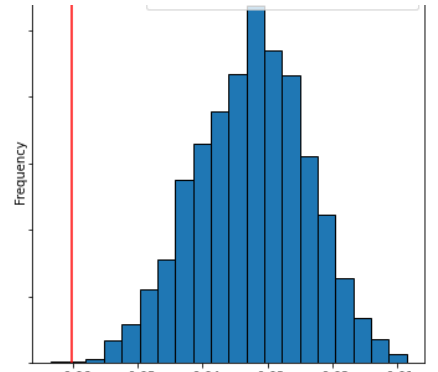


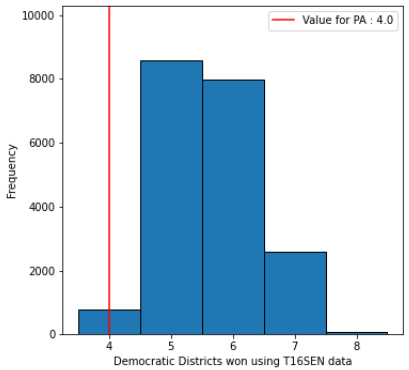


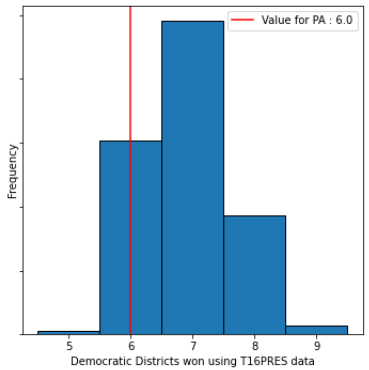


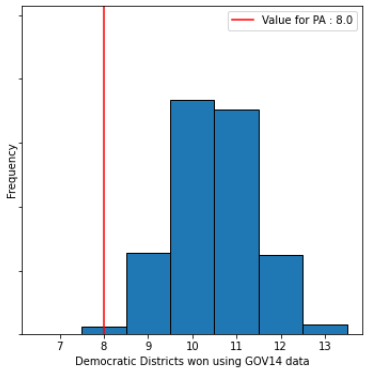


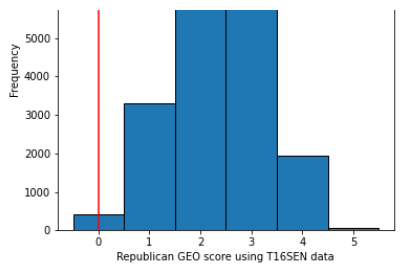


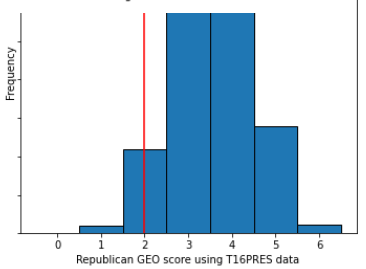


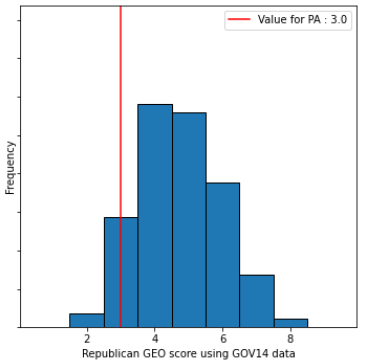








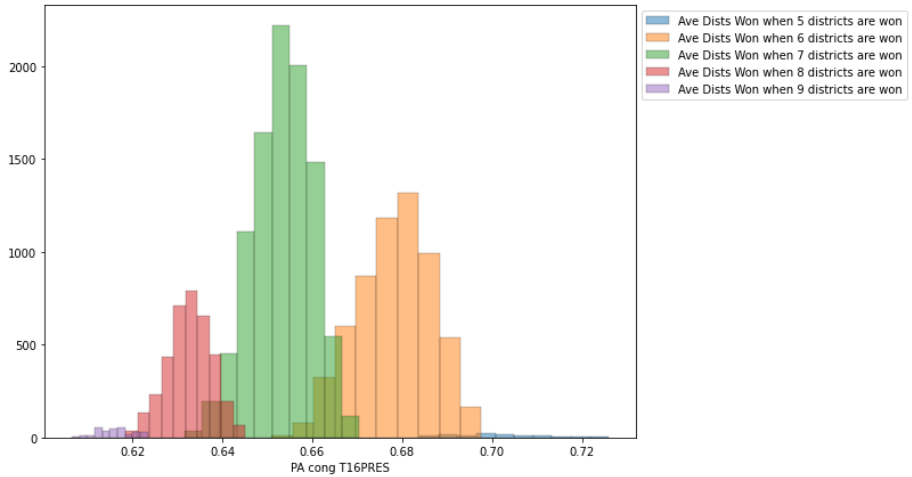


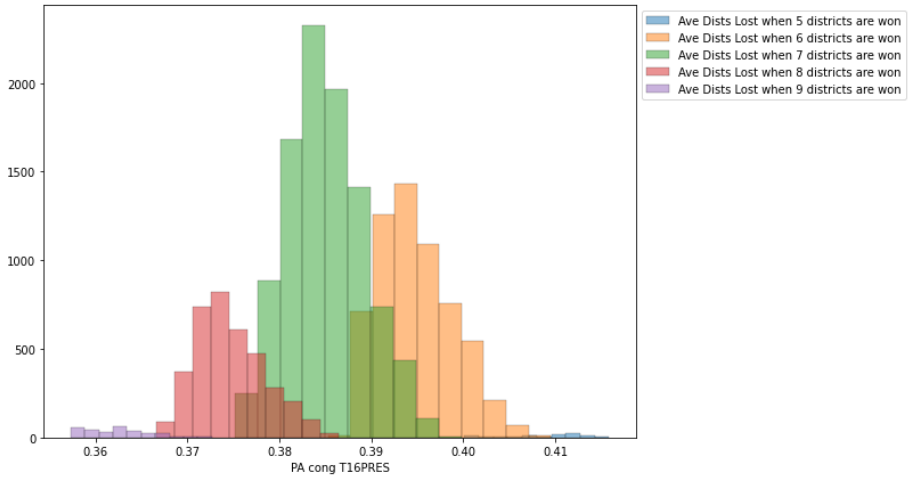


Why do EG, Declination, GEO metric track "districts won" so well?

- For EG:
- For Declination:

$$EG = \left(S - \frac{1}{2}\right) - 2\left(V - \frac{1}{2}\right) + \frac{S(1-S)(1-\rho)}{S(1-\rho)+\rho}$$



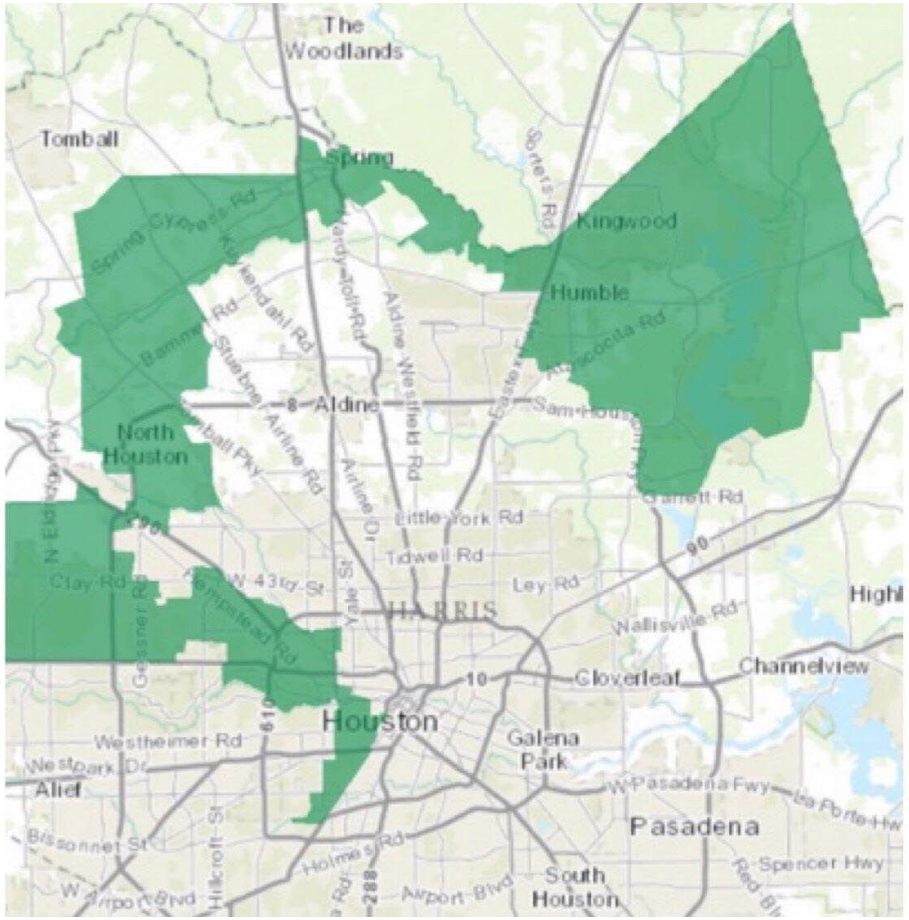


- For GEO: GEO metric counts "additional districts that can become competitive for party A"
  - If party A wins an additional seat, this number likely goes down for party A, up for party B

Conclusions

- Every metric can be gamed
- Most metrics give same information as “districts won” on an ensemble
  - *Except MM and PB!* They are sometimes different!
- Some metrics may tell about other things
  - GEO metric can give information about which districts/regions may be gerrymandered
  - MM and PB tell about partisan symmetry, but *not about extreme maps*
- MM and PB should be used with *extreme caution*
  - *They tell about symmetry, not number of districts won*

Thank you!



Texas's old 2nd congressional district