Quot schemes of curve singularities, Hitchin fibers, and link invariants

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We first review the algebraic geometry of plane curve singularities and their Quot schemes. If R is the coordinate ring of such a singularity, and E is a fixed module over R, then the Quot schemes of the singularity associated with E are certain algebraic varieties whose points parametrize the R-submodules of E that have finite codimension. We then present a new conjecture, joint with Oscar Kivinen, asserting that two different types of Quot scheme for the same plane curve singularity are related by a surprisingly simple 'motivic' change of variables. We emphasize that in many cases, this conjecture is a concrete, but mysterious, combinatorial identity. To conclude, we discuss what our conjecture implies about objects in the Langlands program, such as Hitchin fibrations and orbital integrals, and how it was inspired by calculations in knot theory: specifically, Khovanov–Rozansky link homology.