Non-uniqueness of Leray-Hopf solutions for the 3D fractional Navier-Stokes equations perturbed by transport noise

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For the 3D fractional Navier-Stokes equations perturbed by transport noise, we prove the existence of infinitely many Hölder continuous analytically weak, probabilistically strong Leray-Hopf solutions. In the deterministic case, global existence is known ever since the seminal works by Leray (1934) and Hopf (1951), yet recent results show non-uniqueness via the method of convex integration. In contrast to the active field of regularisation by transport noise, we demonstrate that the convex integration method applies also in the presence of such random perturbations, and derive global-in-time solutions which satisfy the energy inequality pathwise on a non-empty random interval [0, tau].