

SHANNON CAPACITY OF HYPERGRAPHS

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There is a large and important collection of Ramsey-type combinatorial problems, closely related to central problems in complexity theory, that can be formulated in terms of the asymptotic growth of the size of the maximum independent sets in powers of a fixed small (directed or undirected) hypergraph, also called the Shannon capacity. An important instance of this is the corner problem studied in the context of multiparty communication complexity in the Number On the Forehead (NOF) model (and other important instances are the cap set problem in additive combinatorics and the USP capacity problem in the complexity theory of matrix multiplication).

We introduce a general method for lower bounding the Shannon capacity of hypergraphs via combinatorial degenerations, a combinatorial kind of “approximation” of subgraphs that originates from the study of matrix multiplication in algebraic complexity theory (and which plays an important role there) but which we use in a novel way. This allows us to obtain the best-known lower bounds for multiple combinatorial problems, including the corner problem and its application in communication complexity.