Time reversal of spinal processes for linear and non-linear branching processes near stationarity

Sylvie Méléard

We consider a stochastic individual-based population model with competition, traitstructure affecting reproduction and survival, and changing environment. The changes of traits are described by jump processes, and the dynamics can be approximated in large population by a non-linear PDE with a non-local mutation operator. Using the fact that this PDE admits a non-trivial stationary solution, we can approximate the non-linear stochastic population process by a linear birth-death process where the interactions are frozen, as long as the population remains close to this equilibrium. This allows us to derive, when the population is large, the equation satisfied by the ancestral lineage of an individual uniformly sampled at a fixed time T, which is the path constituted of the traits of the ancestors of this individual in past times t T. This process is a time inhomogeneous Markov process, but we show that the time reversal of this process possesses a very simple structure (e.g. time-homogeneous and independent of T). This extends recent results where the authors studied a similar model with a Laplacian operator but where the methods essentially relied on the Gaussian nature of the mutations.