

## The classifying space of the surface category and tropical moduli spaces

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Bordism categories form a fruitful connection between moduli spaces of manifolds and algebraic  $K$ -theory. When taking this perspective one usually considers topologically enriched bordism categories, but in this talk we will focus on the truncated bordism categories  $Cob_d$  where any two diffeomorphic bordisms are identified. These are ordinary 1-categories that do not have moduli spaces of manifolds built in, but we will argue that nevertheless they still contain a lot of interesting information about certain moduli spaces.

Concretely, we will consider the surface category  $Cob_2$  and the subcategory  $C \subset Cob_2$  that contains all bordisms without disks. We show that, after passing to a refinement,  $B(Cob_2)$  is  $S^1$  as conjectured by Tillmann. However, we also show that the classifying spaces truncated bordism categories are not always this simple:  $B(C)$  splits of free infinite loop spaces on suspensions of the moduli spaces of tropical curves, the homology of which is known to grow exponentially.

To prove these theorems we develop tools for computing the classifying space of 1-categories where one has a suitable notion of connected components. As the most fundamental application we are able to show that the category of cospans of finite sets has a contractible classifying space. We hope that the tools used here will also prove useful when working with other ‘cospan-like’ categories.