Mathematics of the Climate Crisis: Extremes and Tipping Points
Monday 1 – Friday 5 November
ICMS, Bayes Centre, 47 Potterrow Edinburgh

Michael Ghil I - *Mathematics for the Climate Crisis: A Grand Unification*

For a world in crisis, climatic and otherwise, a bold step is necessary. This step may well benefit from a deeper insight. Between the nonlinear deterministic view of climate evolution of E.N. Lorenz, who notoriously did not get the Nobel Physics Prize, and the linear stochastic view of Klaus F. Hasselmann, who happily just did, a grand unification is possible. Its mathematical underpinnings are the theories of nonautonomous and random dynamical systems (NDS and RDS). The present talk will introduce the necessary concepts and tools, and provide some synthetic and conceptual-model examples, in particular the random attractor of the Lorenz (1963) convection model (LORA).

Michael Ghil II - *Mathematics for the Climate Crisis: A Few Applications*

In this talk, we apply the concepts and methods of NDS and RDS theory to several climate models of low and intermediate resolution. Special attention will be given to extremes and tipping points. To the extent possible within the allotted time frame, the talk will cover the following topics:
1. Topological tipping points (TTPs) in noise-driven chaotic dynamics and their possible connection with extremes and early warning signals.
2. Tipping points in a model of the wind-driven ocean circulation induced by large-scale climate drift.
3. Sharp transitions in paleoclimate models for smooth orbital forcing.
4. Coexistence of two chaotic attractors in a coupled ocean–atmosphere model of the midlatitudes with ENSO forcing.

Special thanks go to the colleagues from whom I learned all the interesting results to be discussed: N. Boers, G. Charó, M.D. Chekroun, J. Demaeyer, T. Mitsui, S. Pierini, K. Riechers, D. Sciamarella, and S. Vannitsem

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Vera Melina Galfi - *Asymptotic theories for extreme events: Applications of extreme value theory and large deviation theory to climate data*

Mathematical theories help us analyse and understand better complex geophysical systems, like the climate system. In this lecture, I will discuss the application of mathematical methods, based on Extreme Value Theory (EVT) and Large Deviation Theory (LDT), to study extreme events – such as heatwaves, heavy rain, floods, and droughts – in climate data. Extreme events occur on a wide range of temporal – and spatial – scales. Some of them are sudden and short-lived, whereas others are long-lasting, i.e. persistent. With EVT, we have, on the one side, a mathematical framework that describes limit distributions for tails of probability density functions (pdf's) of random variables, providing probability estimates for values more extreme than previously observed. LDT, on the other side, contains limit laws for pdf's of sample averages, providing an approximation for the probability of extreme events more persistent than any previously observed. Furthermore, I will discuss typical spatial anomaly patterns of heatwaves, from the perspective of LDT.

Ulrike Feudel I - *Tipping phenomena and resilience: Examples from ecosystems*

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Many systems in nature are characterized by the coexistence of different stable states for a given set of environmental parameters and external forcing. Examples for such behavior can be found in different fields of science ranging from mechanical or chemical systems to ecosystem and climate dynamics. As a consequence of the coexistence of a multitude of stable states, the final state of the system depends strongly on the initial condition. Perturbations, applied to those natural systems can lead to a critical transition from one stable state to another. Such critical transitions are called tipping phenomena in climate science, regime shifts in ecology or phase transitions in physics. Such critical transitions can happen in various ways: (1) due to bifurcations, i.e. changes in the dynamics when external forcing or parameters are varied extremely slow (2) due to fluctuations which are always inevitable in natural systems, (3) due to rate-induced transitions, i.e. when external forcing changes on characteristic time scale comparable to the time scale of the considered dynamical system and (4) due to shocks or extreme events. We discuss these critical transitions and their characteristics and illustrate them with examples from ecological systems. Moreover, we discuss the concept of resilience, which has been originally introduced by C.S. Holling in ecology, and reformulate it in terms of dynamical systems theory.

Ulrike Feudel II - *Transient chaos in dynamical systems subject to a parameter drift*

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We investigate the impact of a monotonous time-dependent forcing, that changes its strength with a non-negligible rate mimicking a drift in the forcing amplitude. Using a paradigmatic example - a pendulum with a moving pivot - we show that underlying transient chaotic dynamics determines for a large range in parameter space the dynamics of the system. The whole dynamics depends crucially on the relation between the time scale of the internal dynamics of the nonlinear system and the time scale of the drift in the amplitude of the forcing. Applying the drift, a large variability in the dynamics is observed despite that fact that the quasistationary approach based on bifurcation diagrams would exhibit a multitude of coexisting periodic attractors. This behavior can be expected for large classes of multistable systems in which fractal basins of attraction with embedded chaotic saddles occur.
In particular we show that starting on a chaotic attractor it is found that the complexity of the dynamics remains very pronounced even when the forcing amplitude has decayed to rather small values. When after the death of chaos the strength of the forcing is increased again with the same rate of change, chaos is found to revive but with a different history. This leads to the appearance of a hysteresis in the complexity of the dynamics. To characterize these dynamics, the concept of snapshot attractors is used, and the corresponding ensemble approach proves to be superior to a single trajectory description, that turns out to be non-representative.
Furthermore, we show that a number of novel types of tippings can be observed due to the topological complexity underlying the dynamics of highly multistable systems. Tipping phenomena are described here as dramatic changes in the dynamics when a parameter undergoes a drift. Tippings from and into several coexisting attractors are possible, and one can find fractality-induced tipping, as well as tipping into chaos. Tipping from or through an extended chaotic attractor might lead to random tipping into coexisting regular attractors, and rate-induced tippings appear not abruptly as phase transitions, rather they show up gradually when the rate of the parameter drift is increased. Since systems of arbitrary time-dependence call for ensemble methods, we argue for a probabilistic approach and propose the use of tipping probabilities as a measure of tipping. We numerically determine these quantities and their parameter dependence for all tipping forms discussed.

Chris Budd - *Mathematical models for the ice ages*

In this talk, I wll perform a careful analysis of the forced PP04 model for climate change, in particular the behaviour of the ice ages. This system models the transition from a glacial to an inter-glacial state through a sudden release of oceanic carbon dioxide into the atmosphere. This process can be cast in terms of a Filippov dynamical system, with a discontinuous change in its dynamics related to the carbon dioxide release. By using techniques from the theory of non-smooth dynamical systems, I will give an analysis of this model in the cases of both no insolation forcing and also periodic insolation forcing. This reveals a rich, and novel, dynamical structure to the solutions of the PP04 model. In particular, we see synchronized periodic solutions with subtle regions of existence which depend on the amplitude and frequency of the forcing. The orbits can be created/destroyed in both smooth and discontinuity-induced bifurcations. We study both the orbits and the transitions between them and make comparisons with actual climate dynamics.

Joint work with Susan Morupisi, Botswana University of Science and Technology

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Valerio Lucarini - *Multistability in the Climate System: Melancholia States and Noise-induced Transitions*

Hayley Fowler - *Anthropogenic intensification of short-duration rainfall extremes and increasing flood risks*

Short-duration (1 to 3 hour) rainfall extremes can cause serious damage to infrastructure and ecosystems and can result in loss of life through rapidly developing (flash) flooding. Short-duration rainfall extremes are intensifying with warming at a rate consistent with atmospheric moisture increase (~7%/K) that also drives intensification of longer-duration extremes (1day+). Evidence from some regions indicates stronger increases to short-duration extreme rainfall intensities related to convective cloud feedbacks but their relevance to climate change is uncertain. This intensification has likely increased the incidence of flash flooding at local scales, particularly in urban areas, and this can further compound with an increased storm spatial footprint to significantly increase total event rainfall. These findings call for urgent climate-change adaptation measures to manage increasing flood risks, including rethinking the way climate change is incorporated into flood estimation guidance.

Rosalind Cornforth - *Title TBC*

Tim Lenton - *Title TBC*

Peter Cox - *Tipping Points in a rapidly changing climate: results from CMIP6 climate models*

Davide Faranda - *Physics-driven methods for understanding Extreme Events in the Earth System*

How can we understand complex systems, such as the Earth System? Complex is what we perceive as spatially, temporally and dynamically rich. In the Earth System, this richness takes the shapes of turbulent vortices, the rage of a thunderstorm, the exponentially fast diffusion of a virus, an economic crisis and endless others. I will present a physically-informed machine learning approach, which combines notions from statistical physics and dynamical systems theory to devise statistical tools that act as magnifying glasses for complex systems. The first specific question I will tackle is how to determine the number of variables, equations or data that we need to describe a specific event within the system we examine. We will see that one may study turbulent flows (such as the von-Karman flow) in a confined geometry with just three simple dynamical equations, which allow for short-term predictability of the flow. Next, I will focus on studying extremes, or rare events, in the system of interest. Using the Earth's atmosphere as example, we will see that extreme events correspond to specific bricks of the underlying and unknown mathematical geometry of the system, namely unstable fixed points of the dynamics. Equipped with these statistical tools, we can search for the footprint of unstable fixed points in the Earth System and discover their correspondence with extreme events encountered in the everyday life, such as turbulent vortices in the atmosphere, storms, hurricanes or earthquakes. This approach may also be applied to other complex natural systems, such as neurological states of the brain.

Peter Ashwin - *Nonautonomous Systems Approaches to Tipping Points in the Earth System*

Peter Ditlevsen - *The role of stochastic dynamics in climate variability*

The basis for stochastic climate modelling will be presented in recognition of the 2021 Nobelprize in physics awarded to Klaus Hasselmann. The idea of fingerprinting as a technique for attributing causality in a changing climate will be presented. Extensions to the non-linear case of multiple states and tipping points will be introduced and applied to explain the observed past climate. The Fokker-Planck equation associated with the stochastic dynamics will be introduced and applied to the climate record. Statistical Early Warning Signals preceeding tipping points will be derived and inferences from those on the climate dynamics will be drawn.

Anna von der Heydt - *Climate response and climate tipping points: dynamical systems approaches*

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The currently ongoing climate change and the debate about possible measures to be taken to limit the consequences of climate change, requires to know and understand the future response of the climate system to greenhouse gas emissions. The Equilibrium Climate Sensitivity (ECS) - defined as the equilibrium global surface temperature response to a doubling of CO_2 - is a key predictor of climate change and has been estimated from climate models, observational, historical and palaeoclimate data. However, its range as reported in the last IPCC-AR6 report, although somewhat narrowed with respect to previous estimates, remains relatively uncertain and has not changed much from the very first estimates. Even more worrying, the latest versions of the most complex Earth System models (CMIP6) suggest even higher estimates for ECS than previous model generations.

This does not mean that climate system science has not advanced; the climate system shows internal variability on many timescales, is subject to non-stationary forcing and is most likely out of equilibrium with the changes in the radiative forcing. Slow and fast feedbacks complicate the interpretation of geological records as feedback strengths vary over time. In the geological past, the forcing timescales were different than at present, suggesting that the response may have behaved differently. Abrupt transitions associated with tipping elements in climate subsystems have occurred in the past and are likely to occur in the future.

In this lecture I will review the progress made in the theoretical understanding of the climate sensitivity. I will introduce the climate attractor and discuss more general notions of ECS on the attractor that can be useful in understanding the response of a climate state to changes in radiative forcing. For example, different time scales in both forcing and response need to be taken into account, and the general underlying assumption of a time-scale separation should be carefully evaluated. The current ECS defines a linear response; however, a climate state close to a tipping point will have a degenerate linear response to perturbations, which can be associated with extreme values of the ECS.

Alberto Carrassi - *Title TBC*

Niklas Boers - *Critical Transitions in Earth System Models*

It has been suggested that several components of the Earth system may respond abruptly to ongoing gradual increases of atmospheric greenhouse gas concentrations and temperatures. Key examples of these potential tipping elements are the polar ice sheets, the Amazon rainforest, and the Atlantic Meridional Overturning Circulation (AMOC). Here, we investigate these systems from a perspective combining modelling evidence with recent observations. Based on the theory of stochastically forced dynamical systems, characteristic changes in the dynamics of systems approaching bifurcation-induced transitions have previously been introduced. We review these methods and introduce some modifications and extensions, before applying them to observation-based data of the Greenland Ice Sheet melt rates, fingerprints of the strength of the AMOC, and remotely sensed data of the Amazon rainforest and the South American monsoon. Results suggest that these subsystems may indeed approaching bifurcation-induced abrupt transition points in response to ongoing anthropogenic climate change.
