Tensor Categories Minicourse

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Day 1 (Pieter & Sam): To motivate the study of tensor categories, and unitary fusion categories specifically, we give a brief overview of how they play a role in various areas of mathematics, including applications to operator algebras and mathematical physics. We then introduce the definition of a tensor category and give some key examples.

Day 2 (Pieter): The aim of today's lecture is to introduce the notion of a unitary fusion category. This includes introducing a linear structure, allowing us to talk about decomposing objects in our category into simple/irreducible objects. We also discuss duals (or conjugates).

Day 3 (Sam): C*-correspondences and actions of fusion categories.

Day 4 (Pieter): We explain the diagrammatic calculus and introduce Q-systems and Morita equivalence. If time permits, we'll also talk about modular tensor categories.

Day 5 (Sam): Ocneanu compactness.

References for further study:

- Dave Penneys: Lecture notes on higher linear algebra (work in progress). Covers (unitary) tensor categories, fusion categories, and 2-categories. Available here: https://people.math.osu.edu/penneys.2/8110//Math8110Autumn2023.html

- Etingof, Gelaki, Nikshych, Ostrik (EGNO): "Tensor categories" (AMS, 2016). Standard reference in the field. Focusses mainly on algebraic aspects. A preprint version of the book can be found on the authors' websites.

- Müger: "Tensor categories: A selective guided tour" (Revista de la Unión Matemática Argentina, 2009). arXiv:0804.3587. Lecture notes for a mini-course on tensor categories, with many references.

- Müger: appendices to Halvorson, "Algebraic quantum field theory" (2006). arXiv:math-ph/0602036. The goal is providing a proof of the Doplicher-Roberts theorem, which says that every symmetric tensor C*-category is the representation category of a compact (super)group, starting from only basic background knowledge.