Lower Bounds on Lyapunov Exponents for SPDE

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I will present recent work with Hairer, Rosati and Yi establishing quantitative lower bounds for the top Lyapunov exponent of linear PDEs driven by two-dimensional stochastic Navier-Stokes equations on the torus. For both the advection-diffusion equation and the linearized Navier-Stokes equations, we prove that the top Lyapunov exponent is bounded below by a negative power of the diffusion parameter κ, showing that the decay rate cannot be super-exponential.

This result provides the first rigorous lower bound on the Batchelor scale in terms of diffusivity, partially answering a conjecture of Doering and Miles. Our approach introduces the concept of "high-frequency stochastic instability," demonstrating that high-frequency states are unstable under stochastic perturbations. This leads to a Lyapunov drift condition for the H^1 norm over the L^2 norm and quantitative estimates on the decay rate in the diffusivity parameter.

I will discuss the implications of our results for understanding mixing phenomena in fluid dynamics.