

Raphael Stuhlmeier

A discrete Hamiltonian perspective on the classical instabilities of deep-water waves

The stability of surface waves in deep water has traditionally been approached via linearisation, starting with various model equations, such as the nonlinear Schrodinger equation or (without restriction to narrow bandwidth) the Zakharov equation. Owing to the form of the deep-water dispersion relation, energy exchange among surface water waves - and thus the possibility of instability - occurs when four or more wavenumbers are involved.

We shall employ the compact, Hamiltonian description of four-wave interaction due to Krasitskii.

In the usual mathematical sense, instability requires that we start from a solution to a set of equations, and describes the evolution of perturbations to that solution. A handful of such explicit solutions - the monochromatic (Stokes') wave and bichromatic wave train - therefore form the backbone of classical instability results.

The simplest, and best known of these cases is the Benjamin-Feir (BFI) or modulational instability, which will be presented in some detail. This instability involves three waves (one of which is counted twice) in near resonance, a carrier wave and two side bands.

Two other instabilities have also been studied extensively, and involve four distinct waves. Type Ia which begins with a bichromatic basic state and type Ib which adds to the Benjamin-Feir instability an additional non-resonant satellite.

In all cases the entire non linear dynamics can be described by the level lines of a certain Hamiltonian function. This enables the explicit computation of steady state solutions, which have been the object of much recent interest. These steady state solutions characterise the linear instability of the underlying Stokes wave (for the BFI and type Ib) or the bichromatic sea state (Type Ia). The phenomenon of phase-locking (or phase coherence) is seen to be coincident with instability. In such cases the dynamic phase -- a specific combination of the interacting phases -- tends initially to a fixed value regardless of the initial configuration of the system. Moreover, certain heteroclinic orbits are identified as new discrete breather solutions, analogous to the famed Akhmediev breather.

This talk is based on joint work with David Andrade.