# QUANTUM EXTERIOR ALGEBRAS, TORSION-FREE BIMODULE CONNECTIONS, AND THE CACTUS GROUP

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A spectral triple, or a noncommutative manifold in the sense of Connes, naturally gives rise to a first-order differential calculus that generalises the space of one-forms on a Riemannian manifold. Extending this structure to a full differential graded algebra — generalising the classical extension to the de Rham complex — admits an abstract description, but explicit computations remain subtle and technically challenging. In this talk, we show how the bimodule map associated to a torsion-free bimodule connection offers a compact, tractable construction of these extensions. We apply this framework to the Heckenberger–Kolb differential calculi over the irreducible quantum flag manifolds. Remarkably, this leads to an unexpected link with Drinfeld's normalized braiding, which yields representations of the cactus group — rather than the usual braid group — on the category of finite-dimensional $U_q(\frak{g})$--modules. Time permitting, we will discuss extensions of this result beyond the Heckenberger--Kolb setting. (Joint work with Alessandro Carotenuto and Junaid Razzaq)