

# Singularities of convex functions and stability of the pushforward by an optimal transport map

Quentin Mérigot

In this work, in collaboration with Alex Delalande and Guillaume Carlier, we consider a convex function  $\phi$  on the Euclidean space, assumed to be Lipschitz, and the associated application  $T = \nabla\phi$ . This application realizes the (quadratic) optimal transport between any probability measure  $\mu$  and its image  $T_{\#}\mu$ . Even when the map  $T$  is discontinuous, we prove that in the neighborhood of a bounded density with compact support, the application  $\mu \rightarrow T_{\#}\mu$  exhibits a Hölder behavior. This study is motivated by applications of optimal transport in statistics, particularly in the linearized optimal transport (LOT) for barycenter computation. Our proof relies on a new bound on the "size" of the singular set of a convex and Lipschitz function on a bounded domain, refining previous results by Alberti.