Prakash Belkale, University of North Carolina at Chapel Hill

Rigid local systems and the multiplicative eigenvalue problem

Local systems are sheaves which describe the behavior of solutions of differential equations. A local system is rigid if local monodromy determines global monodromy. We give a construction which produces irreducible complex rigid local systems on a punctured Riemann sphere via quantum Schubert calculus and strange duality. These local systems are unitary and arise from a study of vertices in the polytopes controlling the multiplicative eigenvalue problem for the special unitary groups SU(n) (i.e., determination of the possible eigenvalues of a product of unitary matrices given the eigenvalues of the matrices). Roughly speaking, we show that the strange duals of the simplest vertices (which can be inductively determined) of these polytopes give (all) possible unitary irreducible rigid local systems.

We note that these polytopes are generalizations of the classical Littlewood-Richardson cones of algebraic combinatorics. Answering a question of Nicholas Katz, we show that there are no irreducible rigid local systems on a punctured Riemann sphere of rank greater than one, with finite global monodromy, all of whose local monodromies have orders dividing n, when n is a prime number.