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Periodic pencils of flat connections and their p-curvature

Abstract: A periodic pencil of flat connections on a smooth algebraic variety X is a linear family of flat connections

$$\nabla(s_1, ..., s_n) = d - \sum_{i=1}^{r} \sum_{j=1}^{n} s_j B_{ij} dx_i,$$

where $\{x_i\}$ are local coordinates on X and $B_{ij} : X \to \mathrm{Mat}_N$ are matrix-valued regular functions. A pencil is periodic if it is generically invariant under the shifts $s_j \mapsto s_j + 1$ up to isomorphism. I will explain that periodic pencils have many remarkable properties, and there are many interesting examples of them, e.g. Knizhnik-Zamolodchikov, Dunkl, Casimir connections and equivariant quantum connections for conical symplectic resolutions with finitely many torus fixed points. I will also explain that in characteristic p, the p-curvature operators $\{C_i, 1 \le i \le r\}$ of a periodic pencil ∇ are isospectral to the commuting endomorphisms $C_i^* := \sum_{j=1}^{n} (s_j - s_j^p) B_{ij}^{(1)}$, where $B_{ij}^{(1)}$ is the Frobenius twist of B_{ij}. This allows us to compute the eigenvalues of the p-curvature for the above examples, and also to show that a periodic pencil of connections always has regular singularites. This is joint work with Alexander Varchenko.